The Gagne – van Hieles Connection: A Comparative Analysis of Two Theoretical Learning Frameworks

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Abstract

There are striking similarities between the van Hieles' model of levels of understanding geometry and the hierarchical levels of learning developed by Robert Gagne. Van Hieles' model has implications not only for teaching and learning geometry but within other branches of mathematics and science as well. Incorporating Gagne's hierarchical learning principles within the context of various branches of mathematics is been recommended by a large number of mathematics educators. In this paper we compare the two theoretical learning frameworks and their implications in mathematics classes. We designed and implemented a research project to investigate the effectiveness of each model on learning geometry in a P-8 pre-service teacher's environment. Test statistic indicated that there was no significant difference between two theoretical learning models on learning geometry.

Background

The nature of learning mathematics and science requires the learner to develop conceptual knowledge and build skills based on activating and mastery of prior knowledge. As mathematics educator we have profound interest in exploring and examining the implications of cognitive learning theories in teaching and learning mathematics.

To explore, very briefly, the cognitive learning theories it is imperative to instigate ones cognitive development. Jean Piaget studied children's' learning processes for half a century. He identified the following four stages in cognitive development: sensory-motor, pre-operational, concrete operational, and formal operational. Piaget's theory suggests that children need to develop a specific cognitive structure before they will be able to perform such tasks as problem solving or abstract thinking. He claimed that all children pass through the four stages in cognitive development in a specific order. Furthermore, he believed that although children can pass through these stages at different rates, but no child can skip one.

Vygostky claimed that learning occurs when children are tackled with a task that they cannot accomplish alone. Although, the task is not within their capabilities, they can do it with the help of a peer or a teacher. He called this immediate level of development, above one's present level of development, the "zone of proximal development". Vygostky also declared that cognitive development has a direct relation to the input from others. Slavin (1997) states, "Vygotsky believed that higher mental functioning usually exists in

conversation and collaboration among individuals before it exists within the individual" (p. 48). A very important application of Vygosky's theory is the idea of helping the learners with the concepts and skills above their zone of proximal development to guide them to self-discovery. This process is called "scaffolding".

Piaget's theory has been criticized because of his suggestion of fixed sequential developmental stages. Nevertheless, Paige's and Vygostky's theories formed the foundation of the constructivist view of cognitive development. Flavell (1992) writes: "As Piaget correctly taught us, children's cognitive structures dictate both what they accommodate to in the environment and how what is accommodated is assimilated. The active nature of their intellectual commerce with the environment makes them to a large degree the manufacturers of their own development" (p. 998).

Constructivism claims that much of learning originates from inside of the learner. The nature of active and guided discovery learning of constructivism leads the learners to a concept formation strategy, which is a mean of pulling discrete items together into larger conceptual schemes. This strategy calls for learners to examine their information and organize it into concepts and to manipulate those concepts. When the students learn how to form concepts, they will be guided by instructors to move one step further and discover the relation between the formed concepts, which is an effective method of problem solving and further learning, which is one of the main of objectives of education.

Mathematical proficiency is unattainable without mastery of the prerequisite skills and concepts. In the present study we examine and compare the Gagne's Hierarchical Learning scheme and Van Hieles' model.

Gagne's Hierarchical Learning

Robert Gagne (1916-2002) developed Hierarchical Learning, which is identifying prerequisites that should be completed before the learner advances to a higher level of learning. He believed that all learners have to pass through these levels in order and no learner can skip a level. Gagne (1965) states, " There are, however, a number of useful generalizations that can be made about several distinguishable classes of performance change (learning), of which I think there are at least eight"(p. v). According to Gagne the eight different classes of levels in which human beings learn are as follow:

- Signal Learning, the individual learns to make a general response to a signal (involuntary).
- Stimulus Response learning, the individual learns to make a precise response to a stimulus (voluntary).
- Chaining, the individual connects two or more stimulus.
- Verbal Association, the individual learns the chains that are verbal.

- Multiple Discrimination, the individual learns to make different responses to different stimuli.
- Concept Learning, the individual learns to make a common response to a class of stimuli.
- Rule Learning (Principle Learning), the individual learns to make a chain of two or more concepts (rule).
- Problem Solving, the individual learns to think.

Gagne also introduces nine sequence events that have to be included in any effective learning. These events are:

1. Gaining Attention: Stimuli activates receptors
2. Stating the Objective: Creates level of expectation for learning
3. Stimulating the Prior Knowledge: Retrieval and activation of short-term memory
4. Presenting the Information: Selective perception of content
5. Eliciting Performance: Responds to questions to enhance encoding and verification
6. Provide Guidance: semantic encoding for storage in long term memory
7. Providing Feedback: reinforcement
8. Assessing Performance: Retrieval and reinforcement of content as final evaluation
9. Enhancing Retention and Transfer to Other Contexts: Retrieval and generalization of learned skill to new situation

Van Hieles' Level of Understanding Geometry

Pierre van Hiele and Dina van Hiele-Geldof developed a model of learning geometry in late 1950's. Shaughnessy and Burger (1985) state:" The van Hieles were mathematics teachers who met with the same difficulties that we all encounter in presenting formal deduction to geometry students. From classroom observations, the van Hieles felt that the students passed through several levels of reasoning about geometric concepts" (p. 420). The objective of van Hiele's model is for students to advance to the level 4 of the model. Yazdani (2007, p. 40) state the levels of van Hieles' model as follow:

- Level 0: Visualization, students see geometric figures as a whole, but they cannot identify the properties of these figures.
- Level 1: Analysis, student can identify the figures and their properties, but they cannot see the interrelationship between different figures, and they also cannot understand definitions.
- Level 2: Informal Deduction, students can use definition but they cannot construct a proof.
- Level 3: Deduction, students can construct a proof but they cannot understand the rigor of geometrical methods.

- Level 4: Rigor, students understand the geometric methods and generalize the geometric concepts at this level. They are also capable of problem solving.

The model is applied by identifying the level of thinking of the students by engaging them in the conversation about the geometric topics, then designing the instruction for their particular level and helping them to advance to the next level. Jurgensen, et al. (1990, p. T61) state that based on the van Heiles, instruction that is developed according to the following sequence, can lead to a higher level of thinking. The phases of thinking based on the van Hieles are as follow:

1. Inquiry/Information, teacher and student discuss the topic and teacher asks some question from the students.
2. Direct Orientation, properties of figures are investigated experimentally.
3. Explication, students are beginning to form a network of relations regarding the topics being studied.
4. Free Orientation, students work more complex problems independently.
5. Integration, using summaries and reviews students integrate their knowledge about specific topic.

After Phase 5 students advance to the next highest level of learning. The teacher repeats the procedure again at the new level.

Design of the Study

We designed and conducted a study to investigate the effectiveness of Gagne's Hierarchal Principles versus van Hieles' Strategy in learning geometry. The objective of the study was to answer the following questions: Was the Gagne's Hierarchal Principles more effective than van Hieles' Strategy in learning geometry. Fifty-three students participated in this study. The design consisted of two groups two experimental treatment groups. One of the groups received the instruction based on the principle of Gagne's Hierarchal learning theory. The other experimental group received their instruction according to van Hieles' model of learning geometry. The subjects in two groups were administered a pretest at the beginning of the experiment and a posttest after sixteen weeks of instruction. The following null hypothesis was tested:
Null Hypothesis: There is no significant difference between Gagne's Hierarchal learning theory and van Hieles' model on learning geometry.

Participants: Participants were two different classes of junior pre-service P-8 teachers, twenty to thirty seven years old, who were assigned to their classes randomly. These classes were selected based on convenience.
Instruments: The instruments used in this study were selected from the textbook publisher's recommended assessments. The instruments were

consistent with the content information and they were valid because they measured exactly what they were supposed to measure.

Analysis and Results

The following statistical analyses were performed on the data obtained from the pretest and posttest:

1.) A t-test was applied to the scores obtained from the pretests administered to the two groups. The mean scores for the first group and the second group were 20.61 and 21.85 points respectively which indicated that were initially at the same level.

2.) Sixteen weeks after the pretest, a posttest was administered to both groups. A repeated measure t-test for dependent samples was used to compare the results of the pretest and the posttest of the first group. The mean difference (mean gain) was 51.48 points at $\alpha=0.05$. A repeated measure t-test for dependent samples was performed on the pretest and posttest scores of the second group. Here the mean difference (mean gain) was 55.1 points at $\alpha = 0.05$. The comparison of the mean gain for the first group and the mean gain for the second group during the same period indicated a difference of 3.62 points, which was not significant $\alpha = 0.05$. Therefore, there was no significant difference between the two groups. The null hypothesis was retained.

Conclusion and Implications

The results of our study and the analogy between Gagne and van Heiles' Principles indicated that there are striking similarities between the van Hieles' levels of understanding geometry and the hierarchical levels of learning developed by Robert Gagne. The resemblance of the two different models of learning is tabulated in table I. The similarities between Gagne's and van Hieles' required processes in advancing each level are tabulated in tables II.

Table II
Comparison between levels of learning in Gagne's and van Hieles' Model

Gagne's Hierarchical Learning	Van Hieles' levels of understanding geometry
Signal Learning Stimulus Response learning	Visualization
Chaining Verbal Association	Analysis
Multiple Discrimination	Informal Deduction,
Concept Learning Rule Learning	Deduction
Problem Solving	Rigor

Table II
Comparison between Gagne's and van Hieles' required processes in advancing each level.

Gagne's Sequence Events of Learning	Van Hieles' Phases of Thinking
Gaining attention Stating the objective Stimulating the prior knowledge	Inquiry/Information
Presenting the information Eliciting performance	Direct Orientation
Assessing performance	Explication
Enhancing retention	Free Orientation
Transfer to other contexts	Integration

Van Hieles' model has implications not only for teaching and learning geometry but within other branches of mathematics and science as well. Integration of the principles outlined by van Hieles and / or the hierarchical learning of Gagne into instructional design within the context of mathematics courses is recommended.

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References

Flavell, J. (1992). *Cognitive development: Past, present, and future*. Developmental Psychology, 1992, vol. 28, No.6, 998-105.

Gagne, R. (1965). *The conditions of learning*. New York, New York: Holt, Rinehart and Winston, Inc.

Jurgensen, R., et al. (1990). *Geometry*. Boston, Massachusetts: Houghton Mifflin Company.

Shaughnessy, M. & Burger W. (1985). *Spadework prior to deduction in geometry*. Mathematics Teacher. September 1985, 78, 419-428.

Slavin, R., (1997). *Educational psychology: Theory and practice*. Boston, Massachusetts: Allyn and Bacon.

Yazdani, M. (2007). *Correlation between Students' Level of Understanding Geometry According to the van Hieles' Model and Students Achievement in Plane Geometry*, Journal of Mathematical Sciences & Mathematics Education, February 2007,Vol. 2, No. 2