University Preparation of Pre-Service Secondary Geometry Teachers: A Need for Research

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Abstract

This paper describes the research that has been done regarding the preparation of pre-service secondary geometry teachers at the college level and makes a call for help in corresponding future research. Our research aimed to discover and pinpoint what is being done to prepare pre-service geometry teachers to teach at the high school level. Teaching proof and teaching with technology were the two themes in geometry teacher preparation research. The areas of the preparation that are lacking and need further investigation are also discussed.

Introduction

We sought to uncover the literature that has been written related to the preparation of pre-service secondary mathematics teachers (PSMTs) in the area of geometry. By preparation, we mean the classes, background, and education provided in the university setting to help prepare PSMTs to teach high school geometry classes. We found that the preparation of PSMTs in geometry has many gaps in the research. Hence, this paper serves as a call to mathematicians and mathematics educators to focus research in this field. In addition, this paper provides specific areas that have been researched in the PSMT geometry preparation and the gaps that remain.

Usiskin (2002) stated two reasons why geometry is important to teach:

1. Geometry "uniquely" connects mathematics with the real outside world.
2. Geometry "uniquely" enables ideas from other areas of mathematics to be "pictured." (p. 72)

The aforementioned reasons contributed to the impetus of our research on PSMTs' undergraduate education in geometry. If teachers are not prepared to teach their students geometry, their students' knowledge base for further mathematics classes or real world situations will likely be affected. Prospective teachers need to experience learning mathematics with the emphasis on problem solving before they are able to fully understand how to teach mathematics (Stohl, 2005).

Consider Usiskin's (2002) first statement stressing the importance of teaching and learning geometry. His rationale can clearly be seen in even the most elementary examples. Making statements such as "that mirror is the shape of a circle," "that picture is in the shape of a rectangle," or "those two boxes are

congruent" is clearly pointing out examples of geometry in the real world. Also consider the distance between two objects, which we can always find by using the Pythagorean theorem, or finding the volume of a jar. All of the preceding examples demonstrate the use of basic geometric concepts. Of course, real world geometry becomes far more sophisticated, but these examples illustrate how basic geometry is all around us.

Usiskin's (2002) second statement points out how important geometry is, not only to the world around us, but also to other areas of mathematics. For instance, understanding the distributive property can be illustrated to a student using area models. Geometry can be used to concretely illustrate this and other abstract concepts to students. Being that geometry is important for understanding both the real world and other topics in mathematics, we sought to determine how teachers are prepared to teach geometry in their classrooms.

From the literature review we conducted, we found very little research has been done on the preparation a PSMT needs to teach high school geometry effectively. Most of the reported research focused exclusively on the use of technology in the classroom using Dynamic Geometry Software (e.g., Jiang, 2002; Pandiscio, 2005), which is important and will be discussed throughout the paper. However, we were also interested in other aspects of teacher preparation such as the coursework taken by PSMTs and the field experiences provided for PSMTs as part of their undergraduate education. According to Jiang, geometry has always been neglected in the area of mathematics education research. In response, we will try to delve deeper to understand why this is the case and discuss the research that has been done and future research needed to be conducted.

How was our research conducted?

Our review of literature began by using a Google Scholar search with the keywords: preparation, pre-service, geometry, and teachers. This search returned almost 4,000 results. This was narrowed by limiting the search to the current years of 1998 to 2008, which brought the results to 1,110 hits. We wanted to know current research, which is why we narrowed our search to these years. Searching again within these 1,110 results, we removed any results that focused only on elementary teachers. This brought the results to 253. From here, abstracts of the articles were examined and only 15 articles were found as relevant. In order to be relevant, an article had to be related to the university preparation of PSMT in teaching geometry. In order to find additional pertinent articles, we searched through peer-reviewed journals in mathematics education using both North Dakota State University's and Purdue University's online journal libraries. This search returned only four additional sources. We then examined the 19 articles by reading each entire article to ensure it related to PSMTs' preparation to teach geometry. This process deemed four articles unusable, leaving 15 articles for use in our paper.

From the difficulty we encountered during the search for sources that applied directly to our topic, we determined the research conducted in relation to

the preparation of pre-service secondary mathematics teachers in the area of geometry is scant. Throughout our research, we found numerous articles related to the preparation of elementary school mathematics teachers rather than secondary mathematics teachers. While preparing elementary mathematics teachers is extremely important, the preparation of secondary mathematics teachers should not be ignored. Elementary mathematics provides students with a mathematical foundation of shape, area, volume, etc.; however, it is secondary mathematics that more readily develops students' knowledge and prepares them for further learning experiences. For instance, knowledge of proof and deductive reasoning are developed in high school geometry course and PSMTs must be prepared to teach these important mathematical reasoning skills (National Council of Teachers of Mathematics [NCTM], 2000). Hence, secondary mathematics teacher preparation deserves attention by the mathematics education community. Thus, our research did not include those articles that were solely related to elementary teachers' preparation to keep the focus at the secondary level. Our primary purpose was to find out what research has been done and thus, from that, conclude what research, if any, still needs to be conducted in the area or PSMTs' undergraduate preparation to teach secondary geometry.

What preparation is given to pre-service mathematics teachers?

If someone decides to pursue a degree in mathematics education, there are state requirements to be met with a set of core classes that are similar to most states. Along with the required mathematics courses, PSMTs may also need to be admitted into their respective university's teacher education program, which usually occurs during their sophomore or junior year. In addition to keeping above a minimum grade point average designated by the university, the programs usually require a grade of "C" or better in variations of required mathematic classes, which usually include calculus, geometry, linear algebra, probably / statistics, real analysis, and sometimes the history of mathematics. Before the PSMTs are admitted to their respective teacher education program, colleges such as North Dakota State University (NDSU) require they take pre-education classes such as Early Experience, Introduction to Teaching, and Educational Psychology. It is in these classes that the PSMT learns how to write lesson plans, classroom management, classroom practices, and evaluation techniques (NDSU). Once admitted to the teacher education program, they are allowed to take their methods, classroom management, and planning classes. Once those are completed, the PSMT moves on to student teaching.

In relation to these requirements, some pre-service teachers may question why they need to take the advanced mathematics courses they are required to complete as part of their undergraduate education since they will most likely not be directly teaching any of the content they are learning (Author & Author A, under review). However, it is the background knowledge and history the teachers are gaining when they are taking these classes (Henning, 2002). For instance, the PSMTs' future students may inquire about why they are

learning how to write proofs or where the Pythagorean theorem originated. It is expected that the classes the PSMTs complete as part of their undergraduate education will help them accurately and knowledgably answer such questions (Conference Board of Mathematical Science, 2001). Not only do mathematics courses taken by PSMTs provide their students with more information, it also gives the teacher credibility. In turn, the students may have more respect for the teacher in the classroom (Martin, McCrone, Bower, & Dindyal, 1998). Understanding where the content the students are learning originated can also provide some connection for the high school students between their high school mathematics and college mathematics (Martin et al., 1998). While some students may not determine their future career path until they are in college, the high school the students attend could have a definite influence on their decision for a major, and it is there many students develop feelings as to "what they want to be when they grow up." As Dick and Rallis (1991) point out, having a good high school mathematics teacher could definitely influence a high school student to decide to pursue their mathematics career future. This influence could result in students showing an interest in a variety of careers related to mathematics: (a) pursuing a teaching career, (b) majoring / minoring in mathematics, or (c) enrolling in extra (meaning above and beyond university required) mathematics classes during their undergraduate career.

What research has been done related to PSMTs preparation to teach geometry?

General

While the research we found related to the preparation of PSMTs in the area of geometry was scarce, themes emerged in existing research. One of those themes was that researchers (e.g., Lee, 2005; Usiskin, 2002) are studying how to prepare teachers to teach using technology in the classroom through the use of Dynamic Geometry Software (DGS) such as Geometers Sketchpad (GSP) (NCTM, 2000). In fact, teacher preparation in technology was the most prevalent area of research in geometry education. Another theme was teacher preparation in teaching and using proofs in the geometry classroom. The pedagogical issue as to whether there is a right or wrong way to teach proofs to the high school students is the main researched topic in this area. The topics of teacher preparation in technology and preparation in teaching and using proofs in the general area of pre-service teacher preparation were the main themes throughout our research and will be discussed in detail.

Technology Related Results

Research has shown that using DGS is an important part of helping high school students understand geometry (e.g. Jiang, 2002; Strickland, 2005). For example, without being able to actually "see" what students are doing, it is hard for the high school students to envision two intersecting circles or two parallel lines and the angles and slopes of each. According to NCTM (2000) using DGS in a high school geometry classroom will help the students form an all-around

understanding of important geometric ideas and constructions. The research conducted by Winicki-Landman (2001) supported NCTM's view on DGS and, more specifically, about GSP and the effects on student learning. A good point made in Winicki-Landman's research was:

> In [his] experience in mathematics teacher education, in general, mathematics teachers do not expose their students sufficiently to experiences that (a) facilitate genuine mathematical research and problem posing; (b) deal with decision making and ask for strategy choices; and (c) lead to real discussion by elaborating and presenting aloud the students' own ideas and by listening to their colleagues' ideas (p. 36).

He goes on to claim that if any of these students become future mathematics teachers, the "crime" will double, and the pattern will continue and the PSMTs will not be taught or, in turn, teach the correct way to learn mathematics. Thus, their students' knowledge will be severely lacking in content. One way this problem can be curbed is by using DGS in the classroom to help facilitate the discussion and to develop their problem solving skills. It has been found that using DGS will help students develop their strategy choices and leads to constructive discussion (Jiang, 2002).

When high school students and PSMTs begin to use DGS, they have the opportunity to discuss with their group members or fellow classmates what they are investigating and are able to deduce and show their own conjectures (Pandiscio, 2005). From research done by Pandiscio [that also was carried through in others' work (e.g. Jiang, 2002; Winicki-Landman, 2001)], it seems that integrating DGS into a PSMT's education would be critical. Surprisingly, most secondary mathematics education programs do not incorporate learning how to use GSP in the classroom into the teacher education curriculum (da Ponte, Oliveira & Varandas, 2002).

Results of research studies (Jiang, 2002; Pandiscio, 2005) have shown that using DGS in the classroom effectively helps develop students (both college and high school level) deductive skills. Students are able to conjecture and use DGS to show that their theories are true or false, and then justify their reasoning (Pandiscio). It is when GSP is used in relation to proofs that educational issues arise. Researchers (e.g., Jiang; Pandiscio) have a difficult time showing that using DGS to prove something is an effective way of teaching students the importance of proof writing in the geometry classroom. When the students begin to explore an idea using DGS, it seems they believe just showing an example, whether it proves their conjecture true or false, is enough justification. Therefore, students do not always understand the need for a formal proof when they use GSP to illustrate the proof with a "picture" (Pandiscio). The tendency of students to rely on "proof by picture" shows the intricacies that come with teaching proof. Since teaching students the need, as well as how to write an appropriate formal proof, is a critical part of the high school geometry classroom, it is extremely important the teachers themselves are able to convey

to the students the need for proofs and how pictures may guide proofs, yet are not a complete proof without further justification. Teachers need to be able to help their students learn how to competently write a formal proof, which comes from their undergraduate career throughout their mathematics classes. Incorporating technology is an effective way to engage the high school students in learning geometry both individually and with their peers (Jiang). We believe that more research needs to be conducted to further investigate the effects of using DGS in the classroom. The studies that have been done show using DGS is effective in helping students develop their deductive skills, but developing their proof skills is a different story.

Proof based results

How should one begin to teach students to write a formal proof? This is an important question any PSMT should ask him/herself before they even enter their teaching career as learning how to write a proof is one of the major topics learned in high school geometry (NCTM, 2000). One of the major results we found when researching how to teach high school students proof writing skills is getting students involved in mathematical discussions or drawing pictures to visualize a concept is critical (Martin et al., 2003). Martin et al. followed one high school geometry teacher's classroom for four months to determine what pedagogical skills a teacher needs to help students comprehend and write formal proofs to the best of their ability. Throughout those four months, they had a researcher in the classroom almost every day, asking the students and teacher questions. Video-recording and tape-recording were used to capture daily classroom routines. This was the only research study found examining teaching of proofs to secondary students. In the following paragraphs, we discuss the study in greater detail along with the results of the study.

Evidence that proof writing is difficult for students has been consistent throughout current research (Herbst, 2001; Hart, 2002). Yet research done related to the use of pedagogical factors in the high school geometry classroom, such as the best way to learn or teach a high school proof writing skills for the first time, are few and far between (Herbst, 2002). Martin et al. (2003) addressed this need to understand pedagogical factors in their research. They decided to focus on interaction between the students and teacher that helped facilitate the students' success in writing formal geometric proofs. They stated, "In a proof based mathematics class, the teacher's responsibility is to help the students learn to produce mathematically valid arguments that meet the standards of rigor set by the teacher" (p. 99). It is then the students' responsibility to meet the teachers' expectations and try their hardest to understand the concept and importance of satisfactory geometric proofs. The teacher they observed worked well interacting with his students and getting them involved in the proof writing process. He had them draw pictures on the board to illustrate their findings and answer their own questions, and also used a lot of verbal interaction between other students and himself. In doing so, the students became more invested in what they were doing, and were more apt to try harder and to get their

conclusion, instead of getting stuck and just giving up on the assignment at hand.

After examining their data, Martin et al. (2003) concluded that using verbal reasoning in the classroom helps the teacher to track how students are progressing throughout the learning process. When students posed a question about their proof or what was going on in their proof, the teacher would ask the students questions to see if they were able to answer their own question by having it directed back at them. In their words, the students are "playing the game [themselves], rather than watching others play the game" (p. 121). Also, the teacher's use of deductive reasoning, along with instructing the students as they made valiant efforts to construct their proofs, was very effective in helping students to develop arguments and supply suitable justification for their reasoning. Even though the teacher was able to help his students grow mentally in their proof writing capabilities, there is still much knowledge left for the students to attain in that aspect.

Although Martin et al.'s (2003) research was one example of a pedagogical choice that helped students learn proofs, it may not be the only way to help students learn proof skills. Additional research must be done in order to come to some conclusion as to which pedagogical choices teachers learn in their undergraduate career and which ones they decide to apply in their classroom as the most appropriate for constructively helping students learn to write a formal geometric proof. Once we have this research base, recommendations can be made for teacher education programs as per how to support our PSMTs in learning to teach using best practices.

What can be concluded from our research?

As previously mentioned, research related to the preparation of PSMTs in the area of geometry was sparse. Even with the aforementioned literature searches, most of the information found pertained to using technology to help prepare pre-service geometry teachers. Our results informed us there is little research and information about what preparation helps a teacher succeed at effectively teaching students in the geometry classroom. The lack of research serves as an impetus for research in the area of geometry teacher preparation so future generations learn geometry from teachers well versed in the material and the pedagogy of teaching of the material. This extends beyond the current research on integrating geometry in the secondary geometry classroom.

While being prepared to use technology is an important part of geometry teacher preparation, additional preparation is needed for future teachers. Secondary geometry courses are where the students are first introduced to the idea of a formal proof (NCTM, 2000) and it is there the students need to be accurately prepared to use the proof writing skills they develop in their geometry class in their future mathematics career. They should enter high school with a base knowledge of mathematical concepts, and once they reach the high school mathematics classroom, that knowledge should be extended and applied

in different ways. Two questions we asked ourselves throughout the research process we felt needed some thought were: (a) What is the appropriate way to teach a proof? and (b) Do the teachers delve right into a straight forward proof or do they start with a two-column proof and go from there? When the education and background is lacking for a high school mathematics teacher, it directly affects their teaching and their students' success in learning the information (Henning, 2002). Hence, the study of teacher learning and understanding of teaching geometry, including the teaching of proofs, needs to be studied in much greater depth. One study is not enough to base teacher preparation recommendations upon.

There is a need for further research on how to prepare PSMTs to teach in a high school classroom in a manner that allows their students to increase their independent explorations, gain a deeper understanding of mathematical concepts and theory, and develop their "geometric sophistication" (NCTM, 2000, p. 309). Comparative studies, qualitative studies, and quantitative studies (Henning, 2002) are all needed to further understand how to prepare teachers for the geometry classroom. Since geometry is thought to be the most "real world" form of mathematics (NCTM), PSTMs' preparation in the area should not be trivialized. It is in the geometry classroom that students learn their base for deductive reasoning and proof skills (NCTM). In turn, students need the logical reasoning skills taught in geometry in almost every career. Thus, our teachers should have a strong base to teach high school geometry. A call for future research in the area of PSMTs' preparation to teach geometry is strongly advised.

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