Hegel's dialectic as a transition mechanism in the van Hiele level theory

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Abstract

The purpose of this paper is to present and exemplify Hegel's dialectic as a transition mechanism for exploring the transitions between levels within the conceptual framework of the van Hiele level theory of geometric thinking. Herbert Spencer's form of the Hegelian dialectic is specialized and applied to bring consecutive levels of increasing complexity through a process of synthesis for the acquisition of the higher level. Thus, one can see how philosophical methods underpin the interpretive research paradigm in mathematics education.

Background

The van Hiele levels of geometric thinking have been described elsewhere (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1988; Hoffer, 1983; Perdikaris, 2011a; van Hiele, 1986; Wirszup, 1976). They offer a description of student thinking processes in Euclidean geometry which, according to Yazdani (2007b), "has helped mankind to organize, classify, describe, represent, explain, interpret, define, and express in a logical way the world surrounding us." The development of student geometric thinking is a hierarchical procedure of five qualitatively different and continuous levels (Perdikaris, 2011a).

The reification process that changes by verbalization the way of learning at each level into a subject matter at the next level helps to explain the growth of student knowledge. The students learn to mathematise their spontaneous activities (Perdikaris, 1996a). Each level appears as a metatheory of the previous level (Freudenthal, 1973) and includes entelechy, i.e., a possibility which predetermines that the level will reach, with the use of a developmental procedure, a different final form.

The van Hiele level theory of geometric thinking provides the foundational idea that the learning of geometry should begin at the visual level where meaningful elaborations of the fundamental geometric figures take place. It springs from Aristotle's taxonomic schema of five basic classes of natural things that are placed hierarchically (Adler, 1978). Since it presents a paradigmatic approach to geometric thinking it can be used as a tool to advance one's ability to investigate student geometric thinking and problem solving.

The transitions between levels are perhaps the greatest problem and an important research issue since they involve applications of intellectual structures to learning and teaching (Perdikaris, 1994, 1998). The course of development of thinking is influenced primarily by the teaching methods and is a right forward transition between levels (van Hiele, 1986). But, the following research findings contradict this claim.

Carpenter (1980) writes that "although there is an almost a priori logic to the sequence of development described by van Hieles, it is not yet clear that the course of development is as rigit as they propose." Burger and Shaughnessy (1986) note that "students move back-and-forth between levels quite a few times while they are in transition from one level to the next." Fuys et al. (1988) state that transitions between levels seem to take place through an "oscillating process." Molina (199l) argues that students could move easily between their levels of thinking and all levels below that.

These findings indicate that the course of development of thinking between levels is not as rigit as described by van Hieles. It is a dynamic process where student thinking is characterized by frequent transitions between consecutive levels (Perdikaris, 2011b).

Accompanying the model of the levels of geometric thinking, the van Hieles proposed a model of five phases of learning which is a means of enhancing student thinking from one level to the next (van Hiele, 1986; van Hiele-Geldof,1984a). As described in Hoffer (1994) the phases of learning are:

1. Familiarization. The students become acquainted with the working domain.
2. Guided Orientation. The students uncover the links that form relationships.
3. Verbalization. The students become aware of relations that they try to express in words with increasing accuracy. Students learn the technical language of the topic.
4. Free Orientation. The students are able to find their way in a network of relations.
5. Integration. The students build an overview of the subject.

The van Hieles claimed that instruction developed according to this model promotes the transition between levels, i.e., this model is a transition mechanism in the van Hiele theory. But, according to the following findings, the model of the phases of learning is not well-justified to be considered as a transition mechanism.

According to Schoenfeld (1986), the model of phases of learning is quite loose since the pedagogical sequence is vague. Crowley (1987) states that the model has to be further refined through analysis of additional data. Clemens and Battista (1992) note that many issues concerning the model are not clear, including the relation of phases to the subject matter. Ding and Jones (2007) write that "the instructional complexity of the guided orientation phase means that far more research is needed." An attempt by Perdikaris (2004) to operationalize prescriptive procedures of this model by measuring the student group uncertainty does not seem to improve it. It might be added that it is not obvious whether it is necessary to go through every phase of learning.

Methods and Procedures

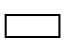
In western philosophy, the dialectic is a method of argument that is traced back to ancient Greek philosophy. The basic idea is present in the philosophy of Heraclitus of Ephesus (550-480 B.C.) who held that nothing is permanent except change. But, the idea of dialectic is found in the older Hindou and Buddhist philosophies (Sriraman and Steinthorsdottir, 2007). It is the result of contradictions (dynamic interplays between unified opposites) which are produced by the struggle of opposites. The dialectic as a resolution of disagreement through rational discussion for the discovery of truth owns its prestige to Plato's dialogues, the best written works that show Socrates philosophy and pedagogical method.

Hegel (1770-1831 A.D.) sees the dialectic as an assertion (thesis) which meets its contradiction or negation (antithesis) and the tension between these opposites is resolved by absorption into a synthesis. A thesis provokes its antithesis and both of them are annulled and preserved in a subsequent synthesis (Williams, 1989). This dialectic looks for a leap of the imagination for elucidating a previous veiled relationship between opposites that were regarded as distinct. The unvarying hop-step-and-jump from thesis to antithesis is a creative method of development whose moving force is the law of unity and struggle of opposites. This law has its roots in the nature of things and is considered as the key to the development of the intellect.

There is an interesting dialectical relationship between "whole" (or totality) and the "parts". The transitions between wholes and parts imply a dynamic process and are a very important issue in philosophy. The parts characterize the whole and the whole is independent of the parts and determines their nature. This back-and-forth (dialectic) is well-characterized by Bahm (1972) where he states that "there are no parts which are not parts of a whole and no wholes which are not wholes of parts. Wholes and parts involve each other; each depends upon the other for being what it is, even though each is not the other."

Hegel's dialectic has been applied to a range of problems. For instance, Hegel used it as a tool for the development of thinking which causes history to unfold, Marx (1818-1903 A.D.), who considered it as the greatest achievement of classical German philosophy, turned it right side up in order to create dialectical materialism and Sriraman and Steinthorsdottir (2007) used it to understand the tension between the notions of excellence and equity in education. Since the van Hiele levels are characterised by an inductive nature, i.e., each level is the product of contradictions implicit in the preceding level, it is proposed that the problem of transitions between levels be explored dialectically.

An interesting form of Hegelian dialectic, offered by Herbert Spencer (1820-1903 A.D.), is exemplified and used as a transition mechanism to resolve the tension between van Hiele levels in order to lead to the acquisition of the higher level. Spencer argues that to attain a thesis directly is often to miss it. In that case, one has to aim at something else which is requisite for the thesis (cited in Weber, I960, pp. 117-118). Our claim is that the indirect target (i.e., this something else) is a characteristic (a special mark, a distinguishing quality) of the lower level which is requisite for the higher level. This form of Hegelian dialectic will be applied as a transition mechanism on the following example taken Crowley (1987).

Consider responses to the questions "What type of figure is this?
 How do you know?" Students at each level are able to respond "rectangle" to the first question. (If a student does not know how to name the figure, he or

she is not at Level 1 for rectangles.) Examples of level-specific responses to the second question are given below. In parenthesis is a brief explanation of the way the statement reflects the assigned level.

Level 1: "It looks like one" or "Because it looks like a door."

(The answer is based on a visual model.)

Level 2: "Four sides, closed, two long sides, two shorter sides, opposites sides parallel, four right angles..." (Properties are listed; redundancies are not seen.)

Level 3: "It is a parallelogram with right angles." (The student attempts to give a minimum number of properties. If queried, she would indicate that she knows it is redundant in this example to say that opposite sides are congruent.)

Level 4: "This can be proved if I know this figure is a parallelogram and one angle is a right angle." (The student seeks to prove the fact deductively.)

Students at Level 1 seek to attain Level 2 (thesis). They try to find the properties that the figure (rectangle) must have, the properties that come out of the figure (necessary conditions). Direct aim at Level 2 is usually problematic since the transitions carry more information than usual at Level 1. Students are feeling for the level but encounter special difficulties and try to get help from the methods of Level 1 (antithesis). Consequently, an indirect course is undertaking that aims at something else. The target can be the quadrilateral (a closed figure with four sides), a characteristic of the rectangle. The quadrilateral is requisite for the rectangle and expressible as a set of verbally stated properties. Manipulations (visual comparisons, distortions, transformations) of the quadrilateral nourish thought at Level 2 and bring the properties into light. Thus, pursuing ends that are requisite for Level 2 acquisition of Level 2 appears unbidden (synthesis).

Students at Level 2 seek to attain Level 3 (thesis). They attempt to find a minimum number of properties (sufficient conditions), the properties that go towards the figure (rectangle) and determine its definition. Direct aim at Level 3 presents difficulties because of the complexity structure of this level. Students have flashes of thinking at Level 3 but return to Level 2 for help (antithesis). In order to attain Level 3 they have to aim at something else. The target can be the characteristic property "opposite sides parallel", the source from which the other properties spring. This property is requisite for a definition of the rectangle as a parallelogram (a quadrilateral with opposite sides parallel) with right angles. The students may appreciate the establishment of a definition and agree with Freudenthal (l973) that "establishing a definition can be an essential feat, more essential than finding a proposition or proof." Thus, pursuing ends other than Level 3 the attainment of Level 3 appears unbidden (synthesis).

Students at Level 3 seek to attain Level 4 (thesis). They try to prove that if this figure is a parallelogram with a right angle, then it is a rectangle (a parallelogram with right angles). Students move back-and-forth between the levels, but do not quite make it and return to Level 3 for help (antithesis). It seems as if their thinking involves ampliative inferences whose content is beyond the available evidence and hence their conclusions are marked by conflict which is the result of ambiguity (Klir & Wierman,1998; Perdikaris, 2004; Voskoglou, 2009). The target can be the special property "right angle" which is a characteristic and requisite for the definition of the rectangle. The rectangle has all the properties of the parallelogram (opposite sides parallel, opposite sides equal, opposite angles equal...). Additionally, if one angle of the parallelogram is a right angle, then all angles are right angles because the adjacent angles of the parallelogram sum up to 180°. Thus, the rectangle is a parallelogram with right angles. This shows that if students pursue ends other than Level 4, then Level 4 appears unbidden (synthesis). Notice that, in the example above, Level 4 was not attained. Mathematics education literature is full of evidence that very few students at secondary schools attain Level 4 (Burger & Shaughnessy, 1986; Fuys et al.,1988).

Results and Conclusions

A specialized Spencer's form of Hegelian dialectic is utilised as a philosophical perspective (source of ideas adapted for a purpose) and a transition mechanism for examining the transitions between van Hiele levels. It is a qualitative method of inquiry whose focus is the content and meaning of any theory and is used here to resolve the tension between consecutive van Hiele levels in order to lead to a forward movement. This is exemplified by applying it as a transition mechanism on an example identifying students' levels of geometric thinking.

This work is exploratory since it examines contradictory and competing ideas and arguments and not confirmatory where a research hypothesis is to be tested against empirical data. This interpretive work is philosophically underpinned by this transition mechanism which is of progressive nature since it brings about stages of complexity through a dialectical process of synthesis. This shows the strong if complex connection between philosophy and pedagogy.

Mathematics education has a domain of inquiry and a body of knowledge regarding the domain. Its methodology (coherent collection of methods) for the acquisition of new knowledge has developed lately in the direction of abandonment of the experimental and descriptive methods which determine the "what works" research paradigm (Lester, 2005).

The new interpretive research paradigm uses philosophical and model-building mathematical methods. It seems that mathematics education and other experimentally based disciplines will develop to a complete maturity if they collaborate with philosophy and mathematics.

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