Ore extension over $\alpha$-quasi-Baer and $\alpha$-p.q.-Baer rings

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Abstract

In this paper we extend some well known results of quasi-Baer and p.q.-Baer to $\alpha$-quasi-Baer and $\alpha$-p.q.-Baer using $\alpha$-weakly rigid ring. Further, we investigate some results for quasi $\alpha$-Armendariz ring and also we give some Examples to illustrate our theory.

Introduction

Throughout this paper $R$ denotes an associative ring with identity, $\alpha$ is an endomorphism of $R$ and $\delta$ an $\alpha$-derivation of $R$, that is $\delta$ is an additive map such that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$, for all $a, b \in R$. Kaplansky [11] introduced Baer rings to abstract various prospects of $AW^*$-Algebra and von-Neumann Algebra. Quasi-Baer rings (i.e. rings in which the right annihilator of every ideal is generated by an idempotent) introduced by Clark [5], are used to characterize when finite dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. The definition of quasi-Baer ring is left-right i.e. a ring $R$ is left (quasi) Baer if and only if $R$ is right (quasi) Baer.

As a generalization of quasi-Baer ring, G.F. Birkenmeier, J.Y. Kim, and J.K. Park [4] introduced the concept of principally quasi-Baer rings. A ring $R$ is called principally quasi-Baer (or right p.q.-Baer) if the right annihilator of a principal right ideal of $R$ is generated by an idempotent. The class of p.q.-Baer ring includes all Baer rings, quasi-Baer rings, abelian p.p. rings and bi-regular rings. Further a number of authors investigated quasi-Baer and p.q.-Baer properties on different structures of a ring.

According to Krempa [12], a monomorphism $\alpha$ of a ring $R$ is called to be rigid if $\alpha\alpha(a) = 0$ implies $a = 0$ for $a \in R$. A ring $R$ is said to be $\alpha$-rigid if there exists a rigid monomorphism $\alpha$ of $R$. Nasr-Isfahani et al. [14] generalized $\alpha$-rigid ring to $\alpha$-weakly rigid ring and used it to transfer the quasi-Baer property and p.q.-Baer property of an $\alpha$-weakly rigid ring $R$ to its extensions such as the skew polynomial ring $R[x; \alpha, \delta]$, skew Laurent polynomial ring $R[x, x^{-1}; \alpha]$, skew power series ring $R[[x; \alpha]]$ and skew Laurent power series ring $R[[x, x^{-1}; \alpha]]$. 
A subset $S$ of a ring $R$ is called $\alpha$-set if $S$ is a $\alpha$-stable set, i.e. $\alpha(S) \subseteq S$. $\alpha$-Baer ring was defined by Han [6] as a ring in which the right annihilator of every $\alpha$-set (resp. $\alpha$-ideal) is generated by an idempotent is called $\alpha$-Baer ring (resp. $\alpha$-quasi-Baer ring). Also a ring $R$ is called right (or left) $\alpha$-p.q.-Baer (resp. right or left p.p.-ring) if the right (or left) annihilator of every right (or left) principal $\alpha$-ideal (resp. $\alpha$-element) is generated by an idempotent. $R$ is called $\alpha$-p.q.-Baer ring (resp. right or left p.p. ring) if it is both right $\alpha$-p.q.-Baer and left $\alpha$-p.q.-Baer. In [6] Han defined and analyzed the behavior of skew polynomial ring over the above mentioned properties for $\alpha$-rigid ring.

In the present article we study the $\alpha$-quasi-Baerness and $\alpha$-p.q.-Baerness for an $\alpha$-weakly rigid ring $R$ and some of its extensions such as skew polynomial ring $R[x;\alpha]$, Ore extension $R[x;\alpha,\delta]$, skew power series ring $R[[x;\alpha]]$ and find some connectedness between an $\alpha$-weakly rigid ring $R$ and its extensions through some results which are a generalization of the results provided in [6], [14]. Further, we also show the same results for a quasi $\alpha$-Armendariz ring $R$.

Ore extension with $\delta = 1$ over $\alpha$-quasi-Baer and $\alpha$-p.q.-Baer ring

In this section we extend the results of [6] to $\alpha$-weakly rigid ring. Further we define the notion of quasi $\alpha$-Armendariz ring as a generalization of quasi-Armendariz ring. Also we prove the same results for quasi $\alpha$-Armendariz ring. Recall from [14] a ring $R$ is called $\alpha$-weakly rigid if for each $a, b \in R, a\alpha(Rb) = 0$ if and only if $aRb = 0$. $\alpha$-weakly rigid ring is a generalization of $\alpha$-rigid ring and $\alpha$-compatible ring. Now we give some examples to show that an $\alpha$-weakly rigid ring $R$ need not to be $\alpha$-rigid.

Example 2.1 Let $Q$ be a ring of rational numbers then $M_2(Q)$ is a prime ring. Suppose $\alpha$ be an automorphism of $M_2(Q)$ which is defined as follows:

$$
\alpha \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}
$$

for each $a, b, c, d \in Q$. Since $M_2(Q)$ is a prime ring and $\alpha$ is an automorphism of $M_2(Q)$, so $M_2(Q)$ is $\alpha$-weakly rigid ring [14, Example...
Now we check that this ring is $\alpha$-rigid or not. Take \( \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in M_2(Q) \), then

\[
\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \alpha \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\]

but \( \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \neq 0 \). Thus \( M_2(Q) \) is not $\alpha$-rigid.

**Example 2.2** We consider a ring \( R \) as

\[
R = \left\{ \begin{pmatrix} a & t \\ 0 & a \end{pmatrix} \mid a \in \mathbb{Z}, t \in \mathbb{Q} \right\}
\]

where \( \mathbb{Z} \) and \( \mathbb{Q} \) are the set of all integers and set of all rational numbers, respectively. Then \( R \) is a commutative ring. Let \( \alpha : R \to R \) be an automorphism of \( R \) defined by

\[
\alpha \left( \begin{pmatrix} a & t \\ 0 & a \end{pmatrix} \right) = \begin{pmatrix} a & t/2 \\ 0 & a \end{pmatrix}.
\]

By [8, Theorem 1], \( R \) is not $\alpha$-rigid. Now suppose any arbitrary \( \begin{pmatrix} a & p \\ 0 & a \end{pmatrix} \) and \( \begin{pmatrix} c & r \\ 0 & c \end{pmatrix} \in R \) such that

\[
\begin{pmatrix} a & t \\ 0 & a \end{pmatrix} \alpha \left( \begin{pmatrix} b & q \\ 0 & b \end{pmatrix} \begin{pmatrix} c & r \\ 0 & c \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
\]

It follows that

\[
\begin{pmatrix} abc & abr + acq + bcp \\ 0 & abc \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
\]

This gives \( abc = 0 \) and \( \frac{abr + acq}{2} + bcp = 0 \), which leads to the following:

1. \( a = 0 \) and \( b = 0 \)
2. \( a = 0 \) and \( c = 0 \)
3. $b = 0$ and $c = 0$
4. $a = 0 = b = c$

By considering that either of the above cases hold valid and true we find

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \begin{pmatrix} q & c \\ 0 & b \end{pmatrix} \begin{pmatrix} r & 0 \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus $R$ is $\alpha$-weakly rigid ring.

For a ring $R$ with an endomorphism $\alpha$, there exists an endomorphism of $R[x; \alpha]$ which extends $\alpha$. For example, consider a map $\overline{\alpha}$ on $R[x; \alpha]$ defined by $\overline{\alpha}(f(x)) = \alpha(a_0) + \alpha(a_1)x + \ldots + \alpha(a_n)x^n$ for all $f(x) = a_0 + a_1x + \ldots + a_n x^n \in R[x; \alpha]$. Then $\overline{\alpha}$ is an endomorphism of $R[x; \alpha]$ and $\overline{\alpha}(a) = \alpha(a)$ for all $a \in R$, that means $\overline{\alpha}$ is an extension of $\alpha$. $\overline{\alpha}$ is called the extended endomorphism of $\alpha$. Here, we shall denote the extended map $\overline{\alpha} : R[x; \alpha] \rightarrow R[x; \alpha]$ by $\overline{\alpha}$.

Now, we begin our main results with Theorem 2.2 but first we give Lemma 2.1 which is required to prove the main Theorems.

**Lemma 2.1** Let $R$ be an $\alpha$-weakly rigid ring and $e \in R$ a right semicentral idempotent of $R$. Then for positive integer $m$,

$$e\alpha^m(r) = e\alpha^m(re).$$

**Proof.** Since $e$ be a right semicentral idempotent of $R$ so $eR = eRe$ which implies $er(1-e) = 0$ for all $e \in R$. It follows that $e\alpha^m(r(1-e)) = 0$ since $R$ be an $\alpha$-weakly rigid ring. Thus $e\alpha^m(r) = e\alpha^m(re)$.

**Theorem 2.2** Let $R$ be an $\alpha$-weakly rigid ring. Then the following conditions are equivalent:

1. $R$ is an $\alpha$-quasi-Baer ring;
2. $R[x; \alpha]$ is a quasi-Baer ring;
3. $R[x; \alpha]$ is an $\alpha$-quasi-Baer ring for every extended $\alpha$-automorphism of $R[x; \alpha]$.

**Proof.** $(1) \Rightarrow (2)$ Suppose $R$ is $\alpha$-quasi-Baer and $I$ be an arbitrary ideal of $R[x; \alpha]$. Consider the set $I_0$ of all the leading coefficients of elements
in $I$ i.e. $I_0 = \{a_n \in R \mid f(x) = a_n x^n + \sum_{i=0}^{n-1} a_i x^i \in I\}$. Then $I_0$ is an ideal of $R$. Note that $I_0$ is an $\alpha$-ideal of $R$, since for
\[ f(x) = a_n x^n + \sum_{i=0}^{n-1} a_i x^i \in I \quad \text{and} \quad g(x) = x \in R, \]
we have
\[ g(x)f(x) = \alpha(a_n)x^{n+1} + \sum_{i=0}^{n-1} \alpha(a_i)x^{i+1} \in I \quad \text{and so} \quad \alpha(a_n) \in I_0. \]
Thus $I_0$ is an $\alpha$-ideal of $R$. Since $R$ is $\alpha$-quasi-Baer, $l_R(I_0) = Re$ for some right semicentral idempotent $e \in R$ which gives $ea_n = 0$ for all $a_n \in I_0$.
Now we show $R[x;\alpha]e = l_{R[x;\alpha]}(I)$. For any
\[ f(x) = a_n x^n + \sum_{i=0}^{n-1} a_i x^i \in I \quad \text{we have} \quad a_n \in I_0, \]
so $ea_n = 0$. Therefore
\[ ef(x) = e(\sum_{i=0}^{n-1} a_i x^i) = ea_n x^n + \sum_{i=0}^{n-1} a_i x^i. \]
Since $ea_n \in I_0$, we get $ea_{n-1} = eea_{n-1} = 0$. Continuing this way we get $ef(x) = 0$ and so
\[ R[x;\alpha]e \subseteq l_{R[x;\alpha]}(I). \]
Suppose $g(x) = \sum_{j=0}^{m-1} b_j x^j \in l_{R[x;\alpha]}(I)$, so for each $f(x) = \sum_{i=0}^{n} a_i x^i \in I$ and $r \in R$,
\[ g(x)f(x) = b_m x^m + \sum_{j=0}^{m-1} b_j x^j f(x) = 0. \]
Therefore $b_m \alpha^m(ra_n) = 0$ for each $r \in R$ which implies that $b_m Ra_n = 0$ since $R$ is $\alpha$-weakly rigid ring, so $b_m = b_m e$. Now
\[ g(x)f(x) = b_m x^m + \sum_{j=0}^{m-1} b_j x^j f(x) = 0. \]
It follows that
\[ \sum_{j=0}^{m-1} b_j x^j f(x) = 0 \]
since
\[ b_m x^m f(x) = b_m e \alpha^m (e) x^m f(x) = b_m e \alpha^m erf(x) = 0 \quad \text{by Lemma 2.1.} \]
In the same way we find that $b_{m-1} = b_{m-1} e$. Continuing this way we get for each $j \ b_j = b_j e$, so $g = ge$ which gives $l_{R[x;\alpha]}(I) \subseteq R[x;\alpha]e$.
Hence $R[x;\alpha]$ is quasi-Baer.

(2) $\Rightarrow$ (3) It is clear.

(3) $\Rightarrow$ (1) Suppose $R[x;\alpha]$ is $\alpha$-quasi-Baer and $\alpha$-weakly rigid.
Let $I$ be any $\alpha$-ideal of $R$. Then by [6, Lemma 1.7] $R[x;\alpha]I$ is an $\alpha$-ideal of $R[x;\alpha]$. Since $R[x;\alpha]$ is $\alpha$-quasi-Baer,
\[ l_{R[x;\alpha]}(R[x;\alpha]I) = R[x;\alpha]e \quad \text{for some idempotent} \quad e(x) = \sum_{i=0}^{n} \in R[x;\alpha]. \]
Thus by [14, Lemma 3.5] $l_{R[x;\alpha]}(R[x;\alpha]I) = l_R(I[x;\alpha] = R[x;\alpha]e(x)$ which implies that $e_i \in l_R(I)$ for all $i$. Again by [14, Lemma 3.5]
Corollary 2.3 ([Theorem 2.3]6) Let \( R \) be a ring with an endomorphism \( \alpha \) and let \( \Lambda_{\alpha} \) be the set of all extended endomorphisms on \( R[x;\alpha] \) of \( \alpha \). If \( R \) is \( \alpha \)-rigid, then the following are equivalent:

1. \( R \) is \( \alpha \)-quasi-Baer;
2. \( R[x;\alpha] \) is quasi-Baer;
3. \( R[x;\alpha] \) is \( \alpha \)-quasi-Baer for all \( \alpha \in \Lambda_{\alpha} \).

Theorem 2.4 Let \( R \) be an \( \alpha \)-weakly rigid ring. Then the following conditions are equivalent:

1. \( R \) is left \( \alpha \)-p.q.-Baer;
2. \( R[x;\alpha] \) is a left p.q.-Baer ring;
3. \( R[x;\alpha] \) is a left \( \alpha \)-p.q.-Baer ring for every extended \(\alpha\)-automorphism of \(R[x;\alpha] \).

Proof. (1) \(\Rightarrow\) (2) Let \( R \) be \(\alpha\)-weakly rigid left \( \alpha \)-p.q.-Baer ring and \( I \) be a left principal ideal of \( R[x;\alpha] \) which is generated by \( h(x) = \sum_{i=0}^{n} h_{i}x^{i} \in R[x;\alpha] \) i.e. \( I = \{ f(x)h(x) \mid f(x) \in R[x;\alpha] \} \). Note that \( I_{0} \) is a left ideal of \( R \) which is generated by \( h_{0}, h_{1}, \ldots, h_{n} \) i.e. \( I_{0} = \{ rh_{i} \mid r \in R \} \). Take \( g(x) = x, \) \( g(x)Rh_{i} = xRh_{i} = \sum_{i=0}^{n} \alpha(Rh_{i})x^{i+1} \) and so \( \alpha(Rh_{i}) \in I_{0} \) for each \( i \).

Thus \( I_{0} \) is an left principal \( \alpha \)-ideal of \( R \). Since \( R \) is \( \alpha \)-p.q.-Baer, \( l_{R}(Rh_{i}) = Re_{i} \) where \( e_{i} \) be right semicentral idempotents of \( R \) therefore \( e_{i}Rh_{i} = 0 \) for all \( i \). Let \( e = e_{i}e_{j} \ldots e_{n} \) which implies \( e \) is also a right semicentral idempotent of \( R \). Thus by [14, Corollary 3.3] \( e \) is a right semicentral idempotent of \( R[x;\alpha] \). We show that \( l_{R[x;\alpha]}(R[x;\alpha]h(x)) = R[x;\alpha]e \). For \( h(x) = \sum_{i=0}^{n} h_{i}x^{i} \in R[x;\alpha] \), \( eh(x) = 0 \) which implies \( ef(x)h(x) = ef(x)eh(x) = 0 \) for any \( f(x) \in R[x;\alpha] \). Therefore \( R[x;\alpha]e \subseteq l_{R[x;\alpha]}(R[x;\alpha]h(x)) \). Again suppose any \( f(x) = \sum_{j=0}^{m} a_{j}x^{j} \in l_{R[x;\alpha]}(R[x;\alpha]h(x)) \). Then
The concept of $\alpha$-skew Armendariz ring has been introduced in [8] which is a generalization of $\alpha$-rigid ring and $\alpha$-Armendariz ring. A ring $R$ is said to be $\alpha$-skew Armendariz ring if for $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j$ in $R[x;\alpha]$ the condition $pq = 0$ implies $a_i\alpha'(b_j) = 0$ for all $i$ and $j$.

The Armendariz property of rings was extended to skew polynomial rings in [10]. Following Hong et al [10], a ring $R$ is called $\alpha$-Armendariz if for $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j$ in $R[x;\alpha]$ the condition $pq = 0$ implies $a_i b_j = 0$ for all $i$ and $j$. $\alpha$-Armendariz ring is a generalization of $\alpha$-rigid ring and Armendariz ring. Hong et al [10] proved that an $\alpha$-Armendariz ring is $\alpha$-skew Armendariz.

In [3] Baser and Kwak introduced the concept of $\alpha$-quasi Armendariz ring. A ring $R$ is called quasi-$\alpha$-Armendariz ring with the endomorphism $\alpha$ (or simply $\alpha$-quasi Armendariz) if for $p(x) = a_0 + a_1 x + \ldots + a_m x^m, q(x) = b_0 + b_1 x + \ldots + b_n x^n$ in $R[x;\alpha]$ satisfy $p(x)R[x;\alpha]q(x) = 0$, implies $a_i R\alpha'(b_j) = 0$ for all $0 \leq i \leq m$ and $0 \leq j \leq n$ or equivalently, $a_i R\alpha'(b_j) = 0$ for any nonnegative integer $t$ and all $i$, $j$. Baser and Kwak [3] also showed that every $\alpha$-quasi Armendariz ring is $\alpha$-skew quasi Armendariz in case that $\alpha$ is an epimorphism, but the converse does not hold, in general. Motivated by [3], Pourtaherian and Rakhimov [15] introduced quasi $\alpha$-Armendariz ring which is a generalization of quasi-Armendariz ring. A ring $R$ is called a quasi $\alpha$-Armendariz (or simply q. $\alpha$-Armendariz) ring if whenever $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j$ in $R[x;\alpha]$ satisfy $pR[x;\alpha]q = 0$, we have $a_i Rb_j = 0$ for all $i$ and $j$. It is easy to see that an $\alpha$-rigid ring is quasi $\alpha$-Armendariz.

Here we refer to an Example from [15] that describes about a quasi $\alpha$-Armendariz ring which is not $\alpha$-rigid.
Given a ring $R$ and a bimodule $R \cdot M$. The trivial extension of $R$ by $M$ is the ring $T(R, M) = R \oplus M$ with the usual addition and multiplication defined as follows:

$$(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2).$$

**Example 2.3** (15, Example 3) Let $R = T(\mathbb{Z}, \mathbb{Q})$ be the trivial extension of $\mathbb{Z}$ by $\mathbb{Q}$, with automorphism $\alpha : R \rightarrow R$ defined by $\alpha((a, s)) = (a, s/2)$. The ring $R$ is quasi $\alpha$-Armendariz but is not $\alpha$-rigid.

Now we show that a quasi Armendariz ring need not be an $\alpha$-weakly rigid ring through the following example.

**Example 2.4** Let $R = \{(a, b) \in \mathbb{Z} \oplus \mathbb{Z} | a \equiv b(\text{mod} \ 2)\}$ be a ring which is commutative reduced ring [9, Example 9], so $R$ is a semiprime ring. Thus $R$ is quasi Armendariz, but not $\alpha$-rigid. Now we check that $R$ is an $\alpha$-weakly rigid ring or not. Define $\alpha : R \rightarrow R$, such that $\alpha((a, b)) = (b, a)$ an automorphism of $R$. Let $(0, 2), (2, 0) \in R$, then

$$(0, 2)\alpha((2, 0)(2, 0)) = (0, 2)\alpha(4, 0) = 0,$$

while

$$(0, 2)(2, 0)(2, 0) = (0, 8) \neq 0.$$

Thus it is clear that $R$ is not $\alpha$-weakly rigid ring.

To prove the results for a quasi $\alpha$-Armendariz ring we need to construct Lemma which is given as follows

**Lemma 2.5** Let $R$ be a quasi $\alpha$-Armendariz ring, then the following conditions hold:

1. If $arb = 0$ then $\alpha^n(a)rb = 0$.

2. If $a\alpha^n(rb) = 0$ then $arb = 0$.

where $a, r, b \in R$ and $m, n$ be some positive integers.

**Proof.** (1) Suppose $arb = 0$ and $f(x) = \alpha(a)x, g(x) = bx \in R[x; \alpha]$. Then

$f(x)rg(x) = (\alpha(a)x)r(bx) = \alpha(a)\alpha(rb)x^2 = \alpha(arb)x^2 = 0$ which implies $\alpha(a)rb = 0$ or $\alpha^n(a)rb = 0$, since $R$ is quasi $\alpha$-Armendariz.
Consider $a \alpha^m(rb) = 0$ and $f(x) = ax^m, g(x) = bx \in R[x; \alpha]$ then $f(x)rg(x) = a \alpha^m(rb)x^{m+1} = 0$. Thus $arb = 0$ since $R$ is quasi $\alpha$-Armendariz.

**Theorem 2.6** Let $R$ be a quasi $\alpha$-Armendariz ring. If $R$ is $\alpha$-quasi-Baer ring then $R[x; \alpha]$ is a quasi-Baer ring.

**Proof.** Let $R$ be a quasi $\alpha$-Armendariz and $\alpha$-quasi-Baer ring, and $I$ be any arbitrary ideal of $R[x; \alpha]$. Consider $I_0$ be the set of all the coefficients of elements of $I$. Observe that $I_0$ is an $\alpha$-ideal of $R$ since for $f(x) = \sum_{i=0}^n a_ix^i \in I$ and $g(x) = x \in R$, we have $g(x)f(x) = \sum_{i=0}^n \alpha(a_i)x^{i+1} \in I$ and so $\alpha(a_i) \in I_0$ for each $i$. Thus $I_0$ is an $\alpha$-ideal of $R$, which gives $l_R(I_0) = Re$ for some right semicentral idempotent $e \in R$ i.e. $ae = 0$ for any $a_i \in I_0$. Now we show that $l_R[x; \alpha](I) = R[x; \alpha]e$. Suppose $f(x) = \sum_{i=0}^n a_ix^i \in I$, so $ef(x) = e(\sum_{i=0}^n a_ix^i) = \sum_{i=0}^n (ea_i)x^i = 0$ so $R[x; \alpha]e \subseteq l_R[x; \alpha](I)$. Again suppose $g(x) = \sum_{j=0}^m b_jx^j \in l_R[x; \alpha](I)$ which implies $g(x)rf(x) = 0$, it follows that $b_jra_i = 0$ since $R$ is quasi $\alpha$-Armendariz. Then $b_j \in l_R(a_i) = Re$ which gives $b_j = b_j e$. Thus $g = ge$ and therefore $l_R[x; \alpha](I) \subseteq R[x; \alpha]e$. Hence $R[x; \alpha]$ is a quasi-Baer ring.

**Theorem 2.7** Let $R$ be a quasi $\alpha$-Armendariz ring. If $R$ is left $\alpha$-p.q.-Baer ring then $R[x; \alpha]$ is a left p.q.-Baer ring.

**Proof.** Let $R$ be quasi $\alpha$-Armendariz left $\alpha$-p.q.-Baer ring and $I$ be a left principal ideal of $R[x; \alpha]$ which is generated by $h(x) = \sum_{i=0}^n h_ix^i \in R[x; \alpha]$ i.e. $I = \{f(x)h(x) \mid f(x) \in R[x; \alpha]\}$. Note that $I_0$ is a left ideal of $R$ which is generated by $h_0, h_1, \ldots, h_n$ i.e. $I_0 = \{rh \mid r \in R\}$. Take $g(x) = x, g(x)h(x) = xh(x) = \sum_{i=0}^n \alpha(h_i)x^{i+1}$ and so $\alpha(h_i) \in I_0$ for each $i$. Thus $I_0$ is an left principal $\alpha$-ideal of $R$.
Since \( R \) is \( \alpha \)-p.q.-Baer so \( l_R(Rh_i) = Re_i \) where \( e_i \) be right semicentral idempotents of \( R \). Let \( e = e_0e_1 \ldots e_n \) which implies \( e \) is also a right semicentral idempotent of \( R \). We show that \( l_{R[x,\alpha]}(I) = R[x;\alpha]e \). For any \( h(x) = \sum_{i=0}^{n} h_i x^i \in R[x;\alpha] \) \( erh = e(\sum_{i=0}^{n} (rh_i)x^i) = \sum_{i=0}^{n} (e = e_0e_1 \ldots e_n)(rh_i)x^i \) which implies \( erh = 0 \). Thus \( R[x;\alpha]e \subseteq l_{R[x,\alpha]}(R[x;\alpha]h(x)) \). Again suppose any \( f(x) = \sum_{i=0}^{n} a_i x^i \in l_{R[x,\alpha]}(R[x;\alpha]h(x)) \). Then \( f(x)R[x;\alpha]h(x) = 0 \) which implies \( f(x)Rh(x) = 0 \) if follows that \( a_j rh_j = 0 \) for all \( r \in R \). Thus \( a_j \in l_R(Rh_j) = Re_i \) which gives \( a_j = a_j e \) so \( f = fe \) and therefore \( l_{R[x,\alpha]}(R[x;\alpha]h(x)) = R[x;\alpha]e \). Hence \( R[x;\alpha] \) is left p.q.-Baer.

Ore extension over \( \alpha \)-quasi-Baer and \( \alpha \)-p.q.-Baer ring

This section discusses about Ore extensions of \( \alpha \)-quasi-Baer and \( \alpha \)-p.q.-Baer rings. In [9] Hong et al. have shown that if \( R \) is an \( \alpha \)-rigid ring, then \( R \) is Baer if and only if \( R[x;\alpha,\delta] \) is a Baer ring. Nasr-Isfahani et al. [14] extended this result for \( \alpha \)-weakly rigid ring to quasi-Baer and p.q.-Baer ring. Here we generalize these results to \( \alpha \)-quasi-Baer and \( \alpha \)-p.q.-Baer ring.

Recall from [13] an ideal \( I \) of a ring \( R \) with an automorphism \( \alpha \) and an \( \alpha \)-derivation \( \delta \) is called an \( (\alpha,\delta) \)-ideal of \( R \) if \( \alpha(I) = I \) and \( \delta(I) \subseteq I \). A ring \( R \) with an automorphism \( \alpha \) and an \( \alpha \)-derivation \( \delta \) is called an \( (\alpha,\delta) \)-quasi-Baer if the left annihilator of every \( (\alpha,\delta) \)-ideal is generated by an idempotent of \( R \).

To prove the main results of this section we need the following Lemma which is a extension of [6, Lemma 1.1].

**Lemma 3.1** Let \( R \) be a ring with an automorphism \( \alpha \) and an \( \alpha \)-derivation \( \delta \). Then

1. If \( I \) is a right \( (\alpha,\delta) \)-ideal of \( R \), then \( RI \) is a right \( (\alpha,\delta) \)-ideal of \( R \);
2. If \( I \) is a left \( (\alpha,\delta) \)-ideal of \( R \), then \( IR \) is a left \( (\alpha,\delta) \)-ideal of \( R \).

**Proof.** It follows from [6, Lemma 1.1].

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Lemma 3.2 Let \( R \) be a ring, \( \alpha \) be an automorphism and \( \delta \) an \( \alpha \)-derivation of \( R \) with \( \alpha \delta = \delta \alpha \). Then the following conditions hold:

1. If \( I \) be an \( (\alpha, \delta) \)-ideal of \( R \) then \( IR[x; \alpha, \delta] \) be an \( (\alpha, \delta) \)-ideal of \( R[x; \alpha, \delta] \);
2. If \( I \) be a right principal \( (\alpha, \delta) \)-ideal of \( R \) then \( IR[x; \alpha, \delta] \) be a right principal \( (\alpha, \delta) \)-ideal of \( R[x; \alpha, \delta] \);
3. If \( I \) be a left principal \( (\alpha, \delta) \)-ideal of \( R \) then \( R[x; \alpha, \delta]I \) be a left principal \( (\alpha, \delta) \)-ideal of \( R[x; \alpha, \delta] \).

For a ring \( R \) with an automorphism \( \alpha \) and \( \alpha \)-derivation \( \delta \) with \( \alpha \delta = \delta \alpha \), there exists an \( \alpha \)-derivation on \( R[x; \alpha, \delta] \) which extends \( \delta \). For example, consider the automorphism \( \alpha \) and the \( \alpha \)-derivation \( \delta \) on \( R[x; \alpha, \delta] \) defined by

\[
\overline{\alpha}(f(x)) = \alpha(a_0) + \alpha(a_1)x + \ldots + \alpha(a_n)x^n
\]
\[
\overline{\delta}(f(x)) = \delta(a_0) + \delta(a_1)x + \ldots + \delta(a_n)x^n
\]

for all \( f(x) = a_0 + a_1x + \ldots + a_nx^n \in R[x; \alpha, \delta] \) and \( \overline{\alpha}(r) = \alpha(r), \overline{\delta}(r) = \delta(r) \) for all \( r \in R \). We shall denote the extended map \( \overline{\alpha} : R[x; \alpha, \delta] \to R[x; \alpha, \delta] \) and \( \overline{\delta} : R[x; \alpha, \delta] \to R[x; \alpha, \delta] \) by \( \delta \), and the image of \( f \in R[x; \alpha, \delta] \) by \( \alpha(f), \delta(f) \), respectively.

Theorem 3.3 Let \( R \) be an \( (\alpha, \delta) \)-weakly rigid ring, \( \alpha \) be an automorphism and \( \delta \) an \( \alpha \)-derivation of \( R \) with \( \alpha \delta = \delta \alpha \). Then the following conditions are equivalent:

1. \( R \) is an \( (\alpha, \delta) \)-quasi-Baer ring;
2. \( R[x; \alpha, \delta] \) is an \( \alpha \)-quasi-Baer ring;
3. \( R[x; \alpha, \delta] \) is an \( (\alpha, \delta) \)-quasi-Baer ring for every extended \( \alpha \)-derivation \( \delta \) of \( R[x; \alpha, \delta] \).

Proof. (1) \(\Rightarrow\) (2) Let \( R \) be an \( \alpha \)-weakly rigid and \( (\alpha, \delta) \)-quasi-Baer ring, and \( I \) be any \( \alpha \)-ideal of \( R[x; \alpha, \delta] \). Suppose that \( I_0 \) be an \( \alpha \)-ideal of \( R \) which is a set of all the leading coefficients of polynomials in \( I \) i.e. \( I_0 = \{ a_n \in R \mid f(x) = a_n x^n + \sum_{i=0}^{n-1} a_i x^i \in I \} \). Now first we show that \( I_0 \) is a \( (\alpha, \delta) \)-ideal of \( R \). Take any \( g(x) = x \in R[x; \alpha, \delta] \) and...
\[ f(x) = \sum_{i=0}^{n} a_i x^i \in I, \]
\[ g(x) f(x) = x f(x) = \alpha(a_n) x^{n+1} + \sum_{i=0}^{n-1} \alpha(a_i) x^{i+1} + \delta(a_n) x^n + \sum_{i=0}^{n-1} \delta(a_i) x^i \in I, \]
which gives \( \delta(a_n) \in I_0 \). Therefore \( I_0 \) is an \( (\alpha, \delta) \)-ideal of \( R \). Since \( R \) is an \( (\alpha, \delta) \)-quasi-Baer ring so \( l_R(I_0) = Re \) for any right semicentral idempotent \( e \in R \) which implies \( eI_0 = 0 \). For \( f(x) = \sum_{i=0}^{n} a_i x^i \in I, \)
\[ ef(x) = e \sum_{i=0}^{n} a_i x^i = 0. \]
Thus \( R[x; \alpha, \delta]e \subseteq l_{R[x; \alpha, \delta]}(I) \). Again suppose \( g(x) = \sum_{j=0}^{m} b_j x^j \in l_{R[x; \alpha, \delta]}(I) \) then \( g(x) f(x) = 0 \) which implies \( b_j ra_i = 0 \) since \( R \) is \( \alpha \)-weakly rigid (from the proof of Theorem 2.2). Then \( b_j \in l_R(a_i) = Re, \) so \( b_j = b_je \) and thus \( g = ge \). Therefore \( l_{R[x; \alpha, \delta]}(I) \subseteq R[x; \alpha, \delta]e \). Hence \( R[x; \alpha, \delta] \) is \( \alpha \)-quasi-Baer.

(2) \( \Rightarrow \) (3) It is straightforward.

(3) \( \Rightarrow \) (1) Similar to Theorem 2.2.

**Corollary 3.4** ([Theorem 3.4][14]) Let \( R \) be an \( \alpha \)-weakly rigid ring. If \( R \) is a quasi-Baer ring then \( R[x; \alpha, \delta] \) is a quasi-Baer ring.

**Corollary 3.5** ([Theorem 3.6][14]) Let \( R \) be an \( \alpha \)-weakly rigid ring. If \( R[x; \alpha, \delta] \) is a quasi-Baer ring then \( R \) is a quasi-Baer ring.

Now we focus on extending the quasi \( \alpha \)-Armendariz property of a skew polynomial rings, as described in section 2, to Ore extension.

**Definition 3.6** A ring \( R \) is called a quasi \( \alpha \)-Armendariz ring if whenever \( p = \sum_{i=0}^{m} a_i x^i \) and \( q = \sum_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta] \) satisfy
\[ pR[x; \alpha, \delta]q = 0, \]
we have \( a_i R b_j = 0 \) for all \( i \) and \( j \).

Here, we introduce the concept of \( (\alpha, \delta) \)-p.q.Baer ring which is a generalization of \( \alpha \)-quasi-Baer, \( (\alpha, \delta) \)-quasi-Baer and \( \alpha \)-p.q.Baer by the following definition:

**Definition 3.7** A ring \( R \) with an automorphism \( \alpha \) and an \( \alpha \)-derivation \( \delta \) is called an \( (\alpha, \delta) \)-p.q.-Baer if the left annihilator of every left principal \( (\alpha, \delta) \)-ideal is generated by an idempotent of \( R \).
**Theorem 3.8** Let $R$ be an $(\alpha, \delta)$-weakly rigid ring, $\alpha$ be an automorphism and $\delta$ an $\alpha$-derivation of $R$ with $\alpha \delta = \delta \alpha$. Then the following conditions are equivalent:

1. $R$ is an $(\alpha, \delta)$-p.q.-Baer ring;
2. $R[x; \alpha, \delta]$ is an $\alpha$-p.q.-Baer ring;
3. $R[x; \alpha, \delta]$ is an $(\alpha, \delta)$-p.q.-Baer ring for every extended $\alpha$-derivation $\delta$ of $R[x; \alpha, \delta]$.

**Proof.** (1) $\Rightarrow$ (2) Let $R$ be an $\alpha$-weakly rigid $(\alpha, \delta)$-quasi-Baer ring and $I$ be a left principal $\alpha$-ideal of $R[x; \alpha, \delta]$ which is generated by $h(x) = \sum_{i=0}^{n} h_i x^i \in R[x; \alpha, \delta]$ i.e. $I = \{ f(x)h(x) \mid f(x) \in R[x; \alpha, \delta] \}$. Note that $I_0$ is a left ideal of $R$ which is generated by $h_0, h_1, \ldots, h_n$ i.e. $I_0 = \{ rh \mid r \in R \}$. Take $g(x) = x^\alpha$, $g(x)rh(x) = x\alpha(h(x)) = \sum_{i=0}^{n} (\alpha(h_i)) x^i + \sum_{i=0}^{n} \delta(h_i) x^i \in I$ and so $\delta(h_i) \in I_0$ for each $i$. Thus $I_0$ is an left principal $(\alpha, \delta)$-ideal of $R$. The proof of the remaining part is similar to Theorem 2.3.

(2) $\Rightarrow$ (3) It is straightforward.

(3) $\Rightarrow$ (1) Similar to Theorem 2.3.

**Corollary 3.9** ([Theorem 3.9]14) Let $R$ be an $\alpha$-weakly rigid ring. If $R$ is a left p.q.-Baer ring then $R[x; \alpha, \delta]$ is a left p.q.-Baer ring.

**Corollary 3.10** ([Theorem 3.11]14) Let $R$ be an $\alpha$-weakly rigid ring. If $R[x; \alpha, \delta]$ is a left p.q.-Baer ring then $R$ is a left p.q.-Baer ring.

Here, we show main results of this section using quasi $(\alpha, \delta)$-Armendariz ring in place of $(\alpha, \delta)$-weakly rigid ring. First, we define quasi $(\alpha, \delta)$-Armendariz ring which is an extension of quasi $\alpha$-Armendariz ring.

**Definition 3.11** A ring $R$ is called a quasi $(\alpha, \delta)$-Armendariz ring if whenever $p = \sum_{i=0}^{m} a_i x^i$ and $q = \sum_{j=0}^{n} b_j x^j$ in $R[x; \alpha, \delta]$ satisfy $pR[x; \alpha, \delta]q = 0$, we have $a_i R b_j = 0$ for all $i$ and $j$. 
Theorem 3.12 Let $R$ be a quasi $(\alpha, \delta)$-Armendariz and $(\alpha, \delta)$-quasi-Baer ring then $R[x; \alpha, \delta]$ is a $\alpha$-quasi-Baer ring.

Proof. Let $R$ be a quasi $(\alpha, \delta)$-Armendariz and $(\alpha, \delta)$-quasi-Baer ring, and $I$ be any $\alpha$-ideal of $R[x; \alpha, \delta]$. Suppose that $I_0$ be an $\alpha$-ideal of $R$ which is a collection of all the coefficients of elements of $I$. Now first we show that $I_0$ is a $(\alpha, \delta)$-ideal of $R$. Take any $g(x) = x \in R[x; \alpha, \delta]$ and $f(x) = \sum_{i=0}^{n} a_i x^i \in I$.

$$g(x)f(x) = xf(x) = \sum_{i=0}^{n} \alpha(a_i) x^{i+1} + \sum_{i=0}^{n} \delta(a_i) x^i \in I.$$  

Thus $\sum_{i=0}^{n} \delta(a_i) x^i \in I$ since $I$ is an $\alpha$-ideal of $R[x; \alpha, \delta]$ so $\delta(a_i) \in I_0$. Therefore $I_0$ is an $(\alpha, \delta)$-ideal of $R$. Since $R$ is an $(\alpha, \delta)$-quasi-Baer ring so $l_R(I_0) = R e$ for any right semicentral idempotent $e \in R$ which implies $eI_0 = 0$. Now to show $l_{R[x; \alpha, \delta]}(I) = R[x; \alpha, \delta]e$. Suppose $f(x) = \sum_{i=0}^{n} a_i x^i \in I$ so $ef = e \sum_{i=0}^{n} a_i x^i = 0$. Thus $R[x; \alpha, \delta]e \subseteq l_{R[x; \alpha, \delta]}(I)$. Again suppose $g(x) = \sum_{j=0}^{m} b_j x^j \in l_{R[x; \alpha, \delta]}(I)$ then $g(x)rf(x) = 0$ which implies $b_j r a_i = 0$ since $R$ is quasi $(\alpha, \delta)$-Armendariz. Then $b_j \in l_R(a_i) = Re$ so $b_j = b_j e$ and thus $g = ge$. Therefore $l_{R[x; \alpha, \delta]}(I) \subseteq R[x; \alpha, \delta]e$. Hence the result follows.

Theorem 3.13 Let $R$ be a quasi $(\alpha, \delta)$-Armendariz and left $(\alpha, \delta)$-p.q.-Baer ring then $R[x; \alpha, \delta]$ is a left $\alpha$-p.q.-Baer.

Proof. Suppose $R$ is a quasi $(\alpha, \delta)$-Armendariz and left $(\alpha, \delta)$-p.q.-Baer ring, and $I$ be any left $\alpha$-ideal of $R[x; \alpha, \delta]$ which is generated by $h(x) = \sum_{i=0}^{n} h_i x^i \in R[x; \alpha, \delta]$ i.e. $I = \{ f(x)h(x) \mid f(x) \in R[x; \alpha, \delta] \}$. Let $I_0$ be the set of all coefficients of elements of $I$. Then $I_0$ be a left $\alpha$-ideal of $R$ which is generated by $h_0, h_1, \ldots, h_n$. Note that $I_0$ is a left $(\alpha, \delta)$-ideal of $R$ by Theorem 2.6. Since $R$ is a left $(\alpha, \delta)$-p.q.-Baer ring, $l_R(Rh_i) = Re_i$ where $e_i$ be semicentral idempotents of $R$ which implies
Let \( e = e_0 e_1 \ldots e_n \) which implies \( e \) is also a semicentral idempotent of \( R \). Now consider any \( h(x) = \sum_{i=0}^{n} h_i x^i \in I \) so
\[
e rh(x) = \sum_{i=0}^{n} er(h_i x^i) = \sum_{i=0}^{n} (e_0 e_1 \ldots e_n) r(h_i x^i) = 0,
\]
since \( e_i \) is a semicentral idempotent of \( R \). Therefore \( R[x; \alpha, \delta] e \subseteq l_{R[x; \alpha, \delta]}(R[x; \alpha, \delta]h(x)) \). Again suppose \( g(x) = \sum_{j=0}^{m} b_j x^j \in l_{R[x; \alpha, \delta]}(R[x; \alpha, \delta]h(x)) \). Then \( g(x)R[x; \alpha, \delta]h(x) = 0 \) which implies that \( g(x)Rh(x) = 0 \).
It follows that \( b_j \in l_{R}(Rh_j) = Re \) which gives \( b_j = b_j e_0 e_1 \ldots e_n \) and therefore \( g = ge \) implies \( l_{R[x; \alpha, \delta]}(R[x; \alpha, \delta]h(x)) \subseteq R[x; \alpha, \delta] e \). Hence the result follows.

**Skeew power series over \( \alpha \)-quasi-Baer ring**

In this section we consider the relationship between the properties of being \( \alpha \)-quasi-Baer of a ring \( R \) and of the skew power series ring \( R[[x; \alpha]] \). Further we introduce the concept of quasi \( \alpha \)-Armendariz of power series type which is an extension of quasi \( \alpha \)-Armendariz ring and also an extension of skew \( \alpha \)-Armendariz property of a ring \( R \) defined in [15].

**Theorem 4.1** Let \( R \) be an \( \alpha \)-weakly rigid ring. Then the following conditions are equivalent:
1. \( R \) is an \( \alpha \)-quasi-Baer ring;
2. \( R[[x; \alpha]] \) is a quasi-Baer ring;
3. \( R[[x; \alpha]] \) is a \( \alpha \)-quasi-Baer ring for every extended \( \alpha \)-automorphism of \( R[[x; \alpha]] \).

**Proof.** (1) \( \Rightarrow \) (2) Suppose \( R \) is \( \alpha \)-quasi-Baer and \( I \) be an arbitrary ideal of \( R[[x; \alpha]] \). Let \( I_0 \) be the set of leading coefficients of elements in \( I \) i.e. \( I_0 = \{ a_n \in R \mid \text{there exists } a_n x^n + \sum_{i=n+1}^{\infty} a_i x^i \in I, \text{ for some non-negative integer } n \text{ and } a_i \in R \} \). Then \( I_0 \) is an ideal of \( R \). Note that \( I_0 \) is an \( \alpha \)-ideal of \( R \) since for \( f(x) = a_n x^n + \sum_{i=n+1}^{\infty} a_i x^i \in I \) and \( g(x) = x \in R \), we have \( g(x)f(x) = \alpha(a_n) x^n \sum_{i=n+1}^{\infty} \alpha(a_i) x^i \in I \) and so \( \alpha(a_i) \in I_0 \) for each \( i \).
Thus \( I_0 \) is an \( \alpha \)-ideal of \( R \), which gives \( l_{R}(I_0) = Re \) for some idempotent
\[ e \in R. \text{ For any } f(x) = \sum_{i=0}^{\infty} a_i x^i \in I \text{ we have } a_n \in I_0, \text{ so } ea_n = 0. \]

Therefore \[ ef(x) = e(\sum_{i=0}^{\infty} a_i x^i) = ea_n x + e(\sum_{i=n+1}^{\infty} a_i x^i). \]

Since \( ea_n \in I_0 \), we get \( ea_n = eea_n = 0 \). Continuing in this way we get \( ef(x) = 0 \) and so \( R[[x; \alpha]]e \subseteq l_{R[[x; \alpha]]}(I) \). Proof of remaining part of this Theorem is similar to Theorem 2.2 Hence \( R[[x; \alpha]] \) is quasi-Baer.

(2) \( \Rightarrow \) (3) It is clear.

(3) \( \Rightarrow \) (1) Similar to Theorem 2.2.

Corollary 4.2 ([Theorem 3.28][14]) Let \( R \) be an \( \alpha \)-weakly rigid ring. If \( R \) is a quasi-Baer ring then \( R[[x; \alpha]] \) is a quasi-Baer ring.

Motivated by Pourtaherian and Rakhimov [15], we define quasi \( \alpha \)-Armendariz ring of power series type as follows:

**Definition 4.3** Let \( R \) be a ring and \( \alpha \) be an endomorphism of \( R \).

Then \( R \) is called a quasi \( \alpha \)-Armendariz ring of power series type if for \( p = \sum_{i=0}^{\infty} a_i x^i, q = \sum_{j=0}^{\infty} b_j x^j \in R[[x; \alpha]] \), \( pR[[x; \alpha]]q = 0 \) implies \( a_i R b_j = 0 \) for all \( i \) and \( j \).

**Theorem 4.4** Let \( R \) be a quasi \( \alpha \)-Armendariz of power series type. If \( R \) is \( \alpha \)-quasi-Baer ring then \( R[[x; \alpha]] \) is a quasi-Baer ring.

Proof. Let \( R \) be a quasi \( \alpha \)-Armendariz of power series type and \( \alpha \)-quasi-Baer ring, and let \( I \) be any arbitrary ideal of \( R[[x; \alpha]] \). Consider \( I_0 \) be the set of all the coefficients of elements of \( I \). Observe that \( I_0 \) is an \( \alpha \)-ideal of \( R \) since for \( f(x) = \sum_{i=0}^{\infty} a_i x^i \in I \) and \( g(x) = x \in R \), we have \( g(x)f(x) = \sum_{i=0}^{\infty} \alpha(a_i)x^{i+1} \in I \) and so \( \alpha(a_i) \in I_0 \) for each \( i \). Thus \( I_0 \) is an \( \alpha \)-ideal of \( R \), which gives \( l_R(I_0) = Re \) for some right semicentral idempotent \( e \in R \) i.e. \( ea_n = 0 \) for any \( a_n \in I_0 \). Now we show that \( l_{R[[x; \alpha]]}(I) = R[[x; \alpha]]e \). Suppose \( f(x) = \sum_{i=0}^{\infty} a_i x^i \in I \), so \( ef(x) = \sum_{i=0}^{\infty} (ea_i)x^i = 0 \) so \( R[[x; \alpha]]e \subseteq l_{R[[x; \alpha]]}(I) \). Again suppose \( g(x) = \sum_{j=0}^{\infty} b_j x^j \in l_{R[[x; \alpha]]}(I) \) which implies \( g(x)rf(x) = 0 \), it follows...
that \( b_j r a_j = 0 \) since \( R \) is quasi \( \alpha \)-Armendariz of power series type. Then
\[ b_j \in l_R(a_j) = Re \] which gives \( b_j = b e \). Thus \( g = ge \) and therefore
\[ l_R[[x;\alpha]](I) \subseteq R[[x;\alpha]]e \]. Hence the result follows.

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