A Fair Target Score Calculation Method for Reduced-Over One day and T20 International Cricket Matches

Rohan de Silva, PhD. †

Abstract

In one day internationals and T20 cricket games, the par score is defined as the runs scored by the team batting first. To win the game, the team batting second has to reach the target score which is one run more than the par score. In these games, a decision has to be reached within the allocated time and if there are interruptions, then the number of balls allocated to one or both teams may be reduced. In such a situation, the target score is currently adjusted using the Duckworth Lewis (D/L) method. However, D/L method delivers unrealistic target scores for certain cases exhibiting its unfairness. In this paper, we analyse and identify the reason behind these unrealistic values and propose a method that calculates a fair target score in all situations. The paper also compares the target scores calculated using our proposed method with that of D/L method for a number of example scenarios. It has been concluded that the proposed method is intrinsically fair and should be considered in place of D/L method.

1 Introduction

One day internationals and T20 matches are interesting forms of the cricket game as they result in an outcome within a day or several hours as compared to the traditional form of cricket known as test cricket. However, if there are interruptions due to rain or other causes, then the innings of one or both teams may be decided to be shortened at any state or time point of the match. This may happen before the beginning of the match, during the innings of the team batting first, during the break between the two innings, during the innings of the team batting second or at several of these instances. Except in the case where the innings of both teams are equally shortened before the beginning of the match, in all other cases, the actual runs scored by the team that batted first has to be adjusted depending on the number of balls available to the team batting second. This adjusted score is called the par score. In order to win the match, the team batting second has to reach the target score which is one run more than the par score (Bradshaw, 2010).

In the past, various methods have been developed to calculate the target score. These methods have been tried out in various International Cricket Council (ICC) matches but the outcomes had not been satisfactory. A full
account of these methods can be found in Duckworth & Lewis (1998). The Duckworth-Lewis (D/L) method (Duckworth & Lewis, 1998) introduced in 1998 took a new direction and it was adopted by ICC in 1999. However, it has been noticed that D/L method can also produce target scores that are questionable (Bhogle, 2001). Hence, the research for a better method of target score calculation continues. In 2002, Jayadevan put forward another method (Jayadevan, 2002) and claimed that it was superior to D/L method. Despite his claim, ICC has not replaced D/L method with Jayadevan’s method (J method).

We discuss D/L method and J method in detail in Section 2 of this paper, identifying the primary weaknesses of them. It was those weaknesses that motivated us to develop the new approach discussed in this paper. As we point out in Section 2 of this paper, no accurate target score calculation method can be developed merely by considering the number of balls lost due to the interruptions together with the state of the match (wickets lost, runs scored and the number of balls delivered) and a set of resources curves.

We gradually develop our method in Section 3 and analyse it in Section 4 of this paper. In Section 5, we show the application of our method in all possible example scenarios. These examples include hypothetical as well as real-world scenarios. We show that our method delivers realistic target scores in all these cases. In Section 6 of the paper, we conclude that, because of the excellent properties of our method, ICC could consider it in place of the D/L method.

2. Review of other methods

The key difference of D/L method to the methods that existed before is that it represents the number of balls available and the wickets in hand as a single quantity called the resources. The designers of D/L method have provided resources tables (Duckworth & Lewis, n.d.) which are used to calculate the par score. The values of available resources provided in the tables are also provided as resources graphs by them (Duckworth & Lewis, 1998). These resources graphs of D/L method are shown in Fig. 1.

The target score calculated by D/L method, entirely depends on the accuracy of these graphs. The resources values of D/L method are the world average values and thus, only point estimates. To be statistically correct, we need to convert them to the corresponding interval estimates and then the D/L graphs would change from curves to bands. As shown in Fig. 1 for the case of wickets in hand equal to 6, the resources curve of the batting partnership of a
strong pair of batsmen should actually lie above the resources curve provided by D/L method, whereas that of a weak pair of batsmen should lie below. Statistically, the confidence that we have in a point estimate is zero and hence, the confidence that we have in the target score calculated by D/L method is also zero. In other words, the target score calculated by D/L method is absolutely meaningless in statistical sense.

Fig. 1: Resources graphs of D/L method. Also shown are the regions of resources curves for strong and weak partnerships when 6 wickets are in hand.

It is now clear that D/L method does not take into account the performance of individual batsmen or the batting line up. But as confirmed by Schwartz et. al. (2006) and Norman & Clarke (2010), the batting line up has a significant impact on outcome of the match. Furthermore, the home team enjoys an advantage of about 16 runs in a one day international match (de Silva, Pond & Swartz, 2001) and the strike rate of a batsman can vary depending on the condition of the pitch (Hermanus, 2010). D/L method fails to recognise the effect of all these on the calculated target score.

J method (Jayadevan, 2002) uses two types of data to calculate the target score. The first set of data, called the target score data, is used to calculate the percentage of the innings left. This is similar to the resources available in D/L method as it decreases when the wickets are fallen or the balls are delivered. The second set of data, called the normal data, is used to find the percentage of innings utilised to the point of interruption. As Jayadevan also estimates these data using the performance of past matches, they are also average values and point estimates. As such, the analysis given above in relation to D/L method equally applies for J method as well.

Since the percentage of resources (in D/L method) or the percentage of innings available (in J method) gradually drops with the number of wickets lost,
they disadvantage a team with a strong batting line up when they play against a team with a weak batting line up. A team with a strong batting line up is much immune to an early collapse of their top order wickets and both D/L method and J method ignore this fact. The opposite happens when Team 1 is a team with a weak batting line up and the interruption happens before they have lost many wickets. Bandulasiri (2008) has shown via simulations that D/L method does not produce a fair outcome most of the time when innings of Team 2 has to be terminated at 30th over due to an interruption.

Since a team can arrange their batsmen in any order they want, the concept of decreasing the resources availability with the increasing number of wickets lost as in D/L method or decreasing the percentage of innings left with the increasing number of wickets lost as in J-method is incorrect. Both these methods assume that the batting skills of the batsmen gradually drop from the first wicket partnership to the last wicket partnership. In fact, winning or losing a game is a team effort and the target score calculation should not include the number of wickets lost as the goal is to win the match losing any number of wickets.

3. Proposed method

As we discussed in Section 2 of this paper, it is impossible to set the target score accurately by using only the past data as performed by D/L method and J-method. Average performance characteristics from the past data can be used only when the innings are shortened before the beginning of the match. This is fair because both teams have to face the same number of balls. Those values need to be augmented with the batting performance characteristics of each individual team if a reduction of the number of balls allocated occurs during the innings. This is because the two teams have to face two different situations. This is the philosophy behind our approach.

In our new approach, called the Planned Performance method, the target score calculation considers the planned performance characteristics (P-characteristics) of the two teams. P-characteristics are specific to each team. This is the key difference in our method. It can be appreciated that many different varieties of such P-characteristics can be used in the target score calculation, but they all will have the same philosophy of dealing with the uncertainty of the calculated target score (de Silva, 2011).

In our approach, each innings is treated as consisting of batting segments separated by periods of interruptions. We group each batting segment
together with the balls lost in the interruption that immediately follows it and call it an absolute segment. Then, the runs scored in the batting segments are transformed to the equivalent runs of their absolute segments. Finally, the equivalent runs of all the absolute segments are added together to find the total equivalent number of runs. This quantity is called the earned performance (EP).

For our calculations, we use two sets of curves or data. The first set of curves is the standard performance characteristics (S-characteristics). S-characteristics are developed by studying the performance of all one day international matches. S-characteristics are used for two purposes. The first purpose is to find the equivalent number of runs of an innings that is shortened before the start. The second purpose is to find the equivalent number of runs when an interruption occurs during an innings. The second set of curves called the P-characteristics are specific to each team and can be developed by each team management or provided by the ICC. Regardless of the provider, the P-characteristics should be fixed and available prior to the beginning of the tournament or the match. If team managements are given the responsibility to develop their P-characteristics, then they can construct them using the performance of their individual players. This is probably better as they may be having new players in the team whose performances at international level are not well known. In such a situation, the team management can estimate their performances by taking into account their first class. However, if the development of the P-characteristics of each team becomes the responsibility of ICC, then they will have to find the average performance of each team using the data of past international matches.

Before we introduce the mathematical equations that are required to calculate the par score, let us define the following:

\( b_{\text{max}} \) - The maximum number of balls in an unshortened innings (i.e. \( b_{\text{max}} \) is equal to 300 for a one day game and 120 for a T20 game).

\( b \) – The ball number. This is the legitimate delivery number. If there are any no balls, wides or free hits, then the runs scored from those deliveries are associated with the first legitimate delivery that follows them. The ball number takes into account the legitimate deliveries that were lost due to the interruptions as well.

\( S(b) \) – The average runs that should be scored by a team in a shortened innings as a percentage of average runs scored by the same team in a full-length innings, if the shortening happens before the start of the innings and not during the innings (S-characteristics).
\(i\) - The index used to indicate the interruption number that led to the shortening of the innings.

\(m\) - Number of interruptions that led to the shortening of the innings of Team 1.

\(n\) - Number of interruptions that led to the shortening of the innings of Team 2.

In addition, we define the following quantities in relation to the innings of Team 1. The corresponding quantities of Team 2 are simply represented by its prime. For example, the corresponding quantity of Team 2 for \(b_{\text{max}}\) of Team 1 is denoted by \(b'_{\text{max}}\).

\(b_{\text{max}}\) - The maximum number of balls allocated to Team 1 at the beginning of a shortened innings.

\(P(b)\) - The percentage cumulative runs that Team 1 plans to score as a function of the ball number (P-characteristics of Team 1).

\(b_{i}\) - The ball after which the \(i\)-th interruption occurs in the innings of Team 1 (\(b_0 = 0\)).

\(\Delta b_{i}\) - Length of the \(i\)-th interruption of Team 1 in balls. This is equal to the number of balls reduced due to the \(i\)-th interruption of the innings of Team 1 (\(\Delta b_0 = 0\)).

\(\Delta \rho_i\) - Corresponding value of \(\Delta b_{i}\) in a full-length innings (i.e., \(\Delta \rho_i = \frac{\Delta b_{i}}{b_{\text{max}}}\)).

\(R_{f}\) - Actual runs scored by Team 1 during the \(i\)-th batting segment.

\(R_{\text{par}}\) - The par score.

\(\Theta_{f}\) - If the innings has to be abruptly terminated due to an interruption or dismissal of all the batsmen, the ball after which this occurs in the innings of Team 1.

\(\Theta_{f}\) - Corresponding value of \(\Theta_{f}\) in a full-length innings (i.e., \(\Theta_{f} = \frac{\Theta_{f}}{b_{\text{max}}}\)).

\(R_{\text{max}}\) - Final score of Team 1.

\(R_{\text{eq}}\) - Equivalent runs in the \(i\)-th absolute segment of Team 1.
\( \text{TER} \) – Total equivalent runs of Team 1.

\( EP \) – Earned performance of Team 1.

\( T_i \) – Run modification factor in the \( i \)-th absolute segment of Team 1 given by

\[
T_i = \frac{F(b_i + \Delta b_i)}{F(b_i)} \quad (1)
\]

Our goal is to modify the runs scored by each team using the \( P \)-characteristics and the \( S \)-characteristics to find the earned values. Consider an innings where the number of balls available for Team 1 is reduced to \( b_{max} \) before the beginning of the innings. Later, during the innings, the number of remaining balls is reduced due to \( m \) interruptions. The innings ended abruptly after ball \( b_{max} \) or the batting team faced all deliveries or was all out before facing all deliveries.

![Fig. 2: Scoring pattern in an innings with a single interruption](image)

In the first batting segment, \( P_0 \) to \( P_1 \) in Fig. 2, Team 1 scores \( R_2 \) from \( b_2 \) balls and when the innings is resumed after the first interruption, the number of balls left is \( b_{max} - b_2 - \Delta b_2 \). As the run modification factor for the first absolute segment is \( T_2 \), the equivalent value of runs scored in the first absolute segment is given by \( EP_2 = T_2 R_2 \).

In the second batting segment (\( P_2 \) to \( P_3 \) in Fig. 2), Team 1 scores \( R_2 \) runs and thus, the equivalent runs scored in the second absolute segment is given by

\[ EP_2 = T_2 R_2 \]
The equivalent runs scored in all subsequent absolute segments in an innings with \( m \) interruptions are calculated in the same manner. Therefore, the total equivalent runs scored by Team 1 in the event of \( m \) interruptions in their innings is

\[
T_{ER} = \sum_{l=2}^{n+1} R_{T_l}^l
\]  

(2)

Since the number of balls available was reduced to \( b_{\text{max}} \) before the beginning of the innings, we have to divide the above quantity by \( S(b_{\text{max}}) \). Thus, the earned performance of Team 1 is given by

\[
BP = \frac{1}{S(b_{\text{max}})} \sum_{l=2}^{n+1} R_{T_l}^l
\]  

(3)

Thus, at the beginning of the innings of Team 2, the par score will be given by

\[
R_{\text{par}} = S(b_{\text{max}}) BP
\]  

(4)

Every time there is a reduction of number of balls allocated to Team 2 due to further interruptions, \( R_{\text{par}} \) has to be recalculated as follows.

After the \( n \)-th interruption of the innings of Team 2,

\[
R_{\text{par}} = \sum_{l=0}^{n} R_{T_l}^l + \frac{1}{T_{l=n+1}^T} \left[ S(b_{\text{max}}) BP - \sum_{i=0}^{n} R_{T_l}^i T_{l=1}^T \right]
\]  

(5)

4 Analysis of the Proposed Method

In the following, we analyse our par score calculation and show that it does not produce a biased par score. Let \( h_l = R_l \) and \( g_l = T_l \) for all \( l = 1, 2, \ldots, m+1 \). Note that \( h_l \) are the actual runs of Team 1 in its absolute segments and \( g_l \) are the transformations based on the percentage planned performance rates of Team 1 in the same absolute segments. Let

\[
G = [g_1, g_2, \ldots, g_m, g_{m+1}] \quad \text{and} \quad H = [h_1, h_2, \ldots, h_m, h_{m+1}].
\]

Let \( X = [X_1, X_2, \ldots, X_m, X_{m+1}] \) be the vector formed by rearranging the elements of \( G \) in ascending order of magnitude and let

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\[ Y = [y_1 \ y_2 \ \ldots \ y_n \ y_{n+1}] \] be the vector formed by the corresponding elements of vector \( H \) respectively. It is clear that \( Y \) can have some elements that are not in the ascending order.

As shown below in the proof, the inner product of \( X \) and \( Y \), \( X \cdot Y \) will be maximum if and only if the elements of \( Y \) are also found to be in the ascending order. As a result, the par score will be maximum when \( x_1 \leq x_2 \leq \ldots \leq x_m \leq x_{m+1} \) as well as \( y_1 \leq y_2 \leq \ldots \leq y_m \leq y_{m+1} \). Since a team cannot predict \( H \) values, and therefore, \( Y \) values before the beginning of the game, there is no way for them to manipulate its P-characteristics to achieve a maximum par score. Furthermore, as the points of occurring and the length of interruptions are impossible to predict, it is not possible for a team to adjust the P-characteristics to gain an advantage. Thus, the par score is really randomised eliminating any bias in the S-characteristics.

Proof:

Let \( X = [x_1 \ x_2 \ \ldots \ x_n \ x_{n+1}] \) and \( Y = [y_1 \ y_2 \ \ldots \ y_n \ y_{n+1}] \) be two vectors with all elements arranged in the ascending order of magnitude.

Let \( X' = [x_1 \ x_2 \ \ldots \ x_n \ x_{n+1}] \) represents the vector consisting of all the elements of \( Y \) in the same order except that the two elements \( y_1 \) and \( y_2 \) are now swapped. Since \( x_j \geq x_k \) and \( y_j \geq y_k \), \( (x_j - x_k)(y_j - y_k) \geq 0 \)

and thus, \( x_1y_1 + x_2y_2 - x_1y_2 - x_2y_1 \geq 0 \)

Therefore, \( X' \cdot Y \geq X \cdot Y \). Now, each time, if we take two elements of \( Y \) that are in the ascending order of magnitude and swap their positions, then the resulting inner product with \( X \) will always be smaller than or equal to the previous inner product. This concludes that if we continue this process until all the elements of \( Y \) are no longer in the ascending order of magnitude forming vector \( Z \), then

\[ X \cdot Y \geq X \cdot Z \]

5 Application Examples

We have investigated the performance of our method for the same example scenarios discussed in (Duckworth & Lewis, n.d.).
Table 1: P- Characteristics (ON - Over Number)

The two P-characteristics shown in Table 1 were calculated using the scoring patterns of 36 unshortened one day internationals played between 2009 and 2012 by Australia and England but not necessarily against each other. The average scoring pattern of all teams P(b) was obtained by using the data of 36 unshortened one day internationals played between 2009 and 2012 by each of the teams, Australia, Bangladesh, England, India, New Zealand, Pakistan, South Africa, Sri Lanka, West Indies and Zimbabwe. The S-characteristics was calculated...
calculated performing a non-linear transformation on P(b). The S-characteristics is also shown in Table 1.

In the following, we consider two hypothetical example scenarios that demonstrate how the Planned Performance method takes the strength of the batting dept of a team into consideration. It also shows the insensitiveness of D/L method to the strength of batting depth of a team.

**Example A:**

In a one-day international, Team1 scored 60 runs losing 7 wickets in the first 20 overs before rain interrupted the match. Team1 lost 10 overs due to the interruption but scored 180 runs losing 9 wickets at the completion of its innings. Team 2 was allocated 40 overs. What is the target score for Team 2 at the start of its innings?

Since there is only one interruption in the innings of Team 1 and no interruption in the innings of Team 2, m=1 and n=0.

We find that

\[ b_{\text{ran}} = 300; \quad b = 120; \]

\[ \Delta_0 = 0; \quad \Delta_2 = 300; \quad \Delta_4 = 40; \]

\[ R = 180; \quad R = 180; \quad \text{and} \quad b'_{\text{ran}} = 240. \]

**Part I**

We assume that Team 1 and Team 2 have selected \( P_1 \) and \( P_2 \) (Table 1) as their P-Characteristics respectively. Using \( P_1 \) characteristics of Table 1, Substituting in equation (1) for \( \beta \) and \( \Delta \beta \),

\[ T_1 = \frac{F(180)}{F(120)} = 0.6631 \]

Substituting all the above values in equation (3), we have, \( EP = 211.46 \)

Also, from the S-Characteristics of Table 1, \( S(b_{\text{ran}}) = S(240) = 0.3678 \).

Substituting for \( EP \) and \( S(b_{\text{ran}}) \) in equation (4), we have,
If we assume that Team 1 and Team 2 have selected $P_2$ and $P_1$ as their P-characteristics respectively, then, we will obtain

$$R_{team} = 183$$ (rounded to nearest integer).

Thus, the target score is 183 runs and it is only slightly higher than the previous target score. In this case, Team 1 has selected $P_2$ as its P-characteristics that does not represent a strong batting depth as $P_1$, but in their actual innings they have shown a strong batting depth. As this is contradictory to the chosen P-characteristics, the target score is slightly lower than the previous target score.

If we apply D/L method, since 7 wickets are lost, the reduction of overs will take away only 0.6 (21.8-21.2) percent of resources of Team 1, meaning that Team 1 has used 99.4 (100-0.6) percent of resources. Thus, Team 2 has to only score $1+180(89.3/99.4)$ or 163 runs. This is very unfair because Team 2 knows from the beginning that they have to score only 163 runs to win the match in 40 overs and they can plan well and reach this target easily. Also, this score is much smaller than the actual runs scored by Team 1 in 40 overs.

Example B:

In another one-day international, Team 1 scored 120 runs losing 1 wicket in the first 20 overs when rain interrupted the match. They lost 10 overs due to the interruption and were all out for 180 runs in the 38th over. Team 2 was allocated 40 overs. What is the target score for Team 2 at the start of its innings?

The solution to Example B is similar to that of Example A and is omitted here due to lack of space. But the target scores of Examples A and B are shown in the first two rows of Table 2. The last four rows of Table 2 show the results of our method for the examples (Example 1 to Example 6) given in Duckworth & Lewis (n.d.), The calculation steps of these examples are also omitted here.

The values in column 2 of Table 2 were obtained by assuming that Team 1 and Team 2 have selected P-values of $P_1$ and $P_2$ of Table 1 respectively. The values in column 3 of Table 2 were obtained by interchanging the P values of Team 1 and Team 2.
Table 2: Comparison of Target Score values of Planned Performance method and D/L method

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Target Score of Planned Performance Method</th>
<th>Target Score of D/L method</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Team 1 uses P1 and Team 2 uses P2</td>
<td>Team 1 uses P2 and Team 2 uses P1</td>
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<td>183</td>
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<tr>
<td>Example B</td>
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<td>210</td>
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<tr>
<td>Example 1</td>
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<td>Example 6</td>
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</table>

6 Conclusion

D/L method that is currently being used for target score calculation in limited over and T20 cricket matches delivers unrealistic target scores in some situations. In this paper, we have performed an in-depth analysis and identified the reasons for this behaviour of the D/L method. It has been shown that the fundamental problem in D/L method is the use of curves or tables produced using past statistics of all teams of the world that can only be valid for a game between two world average teams. To alleviate this problem, we have developed a new method that is presented in this paper. Our approach is completely different to the approaches in D/L method and J-method. Our method takes the performance pattern of each of the two teams that are in contest into account and augment the past characteristics using them. These performance patterns can be provided by the individual teams or ICC. This augmentation of past characteristics using the performance patterns gives an improved target score, an outcome that could be more acceptable and fair to both teams and the spectators. Though we have only considered one day internationals, our method can be easily adopted for T20 matches by using the appropriate P-characteristics and S-characteristics. We hope that ICC will consider employing our method in the one-day international and T20 cricket matches.

†Rohan de Silva, PhD, CQUniversity Sydney, Sydney, NSW, Australia
References


